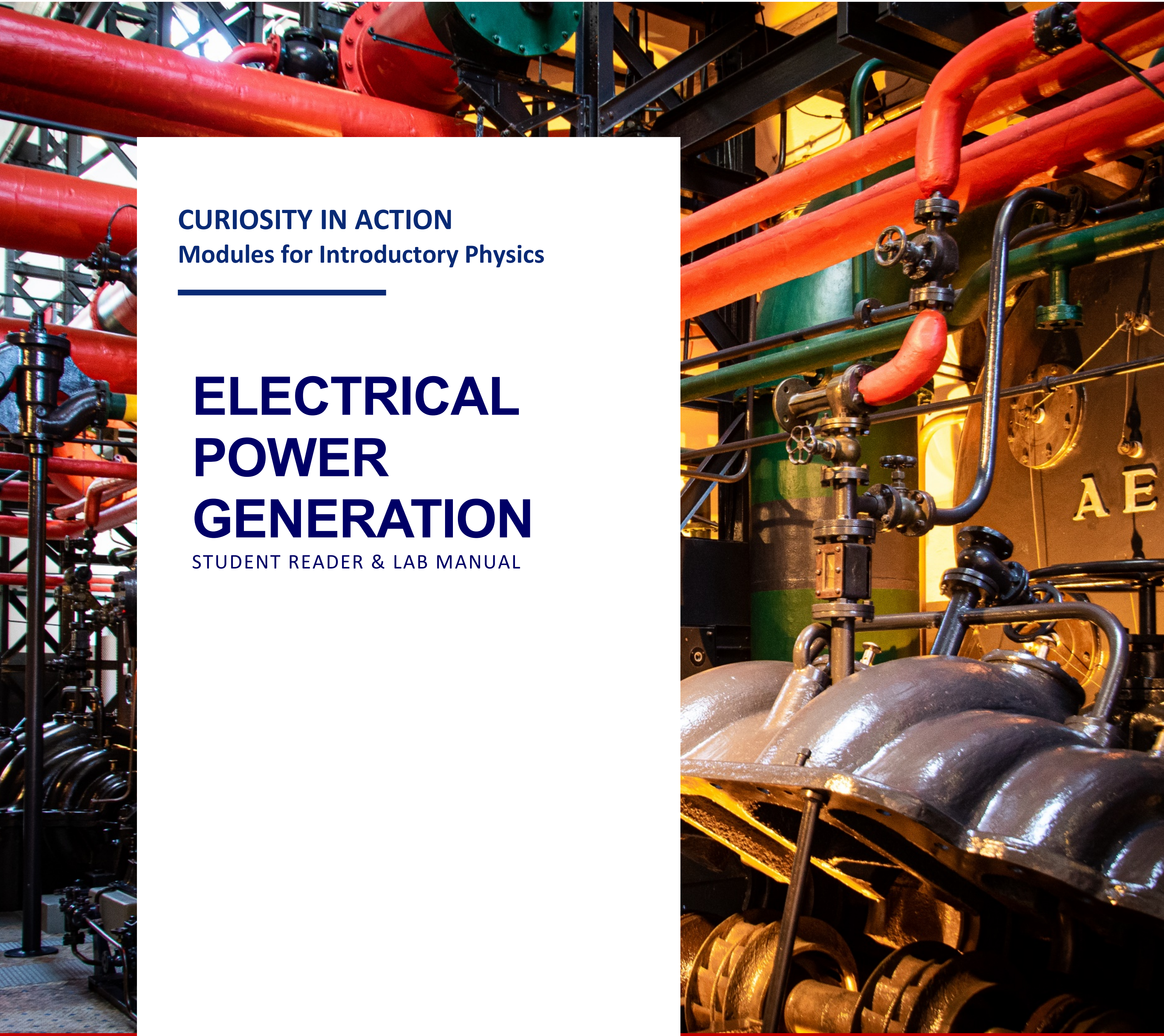

CURIOSITY IN ACTION
Modules for Introductory Physics

ELECTRICAL POWER GENERATION

STUDENT READER & LAB MANUAL

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Electrical Power Generation

In this unit, we will explore how electricity is generated and transmitted across the country. The focus of our study will be the electromagnetic generator, which converts kinetic energy into electrical energy. To understand how these devices work, we need to learn more about the relationship between electricity and magnetism. The final project for this unit will be the construction of a miniature electrical generator designed to meet given specifications.

PHYSICS CONTENT

- Electric Fields & Forces
- Magnetic Fields & Forces
- Electromagnetism
- Faraday's Law of Induction

SCIENCE & ENGINEERING PRACTICES

- Developing & Using Models
- Using Mathematics & Computational Thinking
- Planning & Carrying Out Investigations
- Constructing Explanations & Designing Solutions

CROSS-CUTTING CONCEPTS

- Systems & System Models
- Patterns
- Energy & Matter

Version 1.2 (July 2021)

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Reading: Invention of AC Power Generation

This reading is an excerpt from an article by Jack Foran called "The Day They Turned the Falls On: The Invention of The Universal Electrical Power System"[1]

The name is synonymous with power. Niagara Falls. The natural phenomenon from time immemorial. The myriad tons of water crashing over the solid limestone cliff, with a force that reduces the limestone to boulders, the boulders to rubble, the rubble to a silt the unabated torrent seizes and carries off down the canyon it has formed and shaped through the eons in this same violent, patient manner. The perpetual thunder. The constant cloud of mist from the plummeting waters.

Figure 1: Niagara Falls (Image credit: Wikipedia)

And in the present time, the man-made complement to the natural marvel. The gargantuan electrical power production facilities--the largest in the world--harnessing the prodigious elemental forces.

But five years before start-up of the first large-scale power project at the falls, the method of production and distribution of the power was still undecided. The huge project was to include transmission to Buffalo. Electricity--a novel technology at the time--was only one suggestion. The other methods under consideration were pneumatic, hydraulic, and good old-fashioned mechanical (compressed-air or water mains or steel cables on posts and pulleys the 22-mile distance from Niagara Falls to Buffalo).

The new technology won out in the end. By 1895 the Niagara Falls Power Company began generating alternating current (AC) from three 5000-horsepower generators. The next year electricity was successfully transmitted to Buffalo. The Niagara Falls project ushered in the second phase of the Industrial Revolution and shaped and determined the way power would be produced and delivered from then on.

But in 1890 George Westinghouse recommended that the best way to transport Niagara Falls power to Buffalo would be by compressed air. Westinghouse was likely to know. As the inventor of the air brake, he was the acknowledged expert on pneumatic systems. And of late he had turned his attention to electricity. In 1886 he had organized the Westinghouse Electric Company. By 1890, the company was operating 300 central generating stations.

The Westinghouse organization predominantly utilized AC, and Westinghouse was the champion of AC in the so-called War of the Currents then raging between proponents of AC and advocates of continuous or direct current (DC). But the problem with AC was that it lacked a practical and efficient motor. The AC systems then in operation were primarily lighting systems. In 1888, Nikola Tesla had patented an idea for an AC motor, and Westinghouse promptly bought up the patents and was working on developing the motor. But the motor wasn't ready yet.

[1] Published by the National Center for Case Study Teaching in Science (http://ublib.buffalo.edu/libraries/projects/cases/niagara.htm)

Niagara power--on the scale that it would have to be developed for the project to make sense--would be mainly for industry. For power more than light.

The fierce and stubborn champion of DC was Thomas Edison. Edison had been in the electrical business since the late 1870s, and within a decade was operating in the neighborhood of 1500 generating stations, including isolated plants associated with individual factories or other commercial installations, as well as central stations, supplying electricity to the public at large. The DC systems were basically lighting systems, too, but there was a DC motor for street-rail traction, and DC motors were beginning to be used for various manufacturing purposes. …

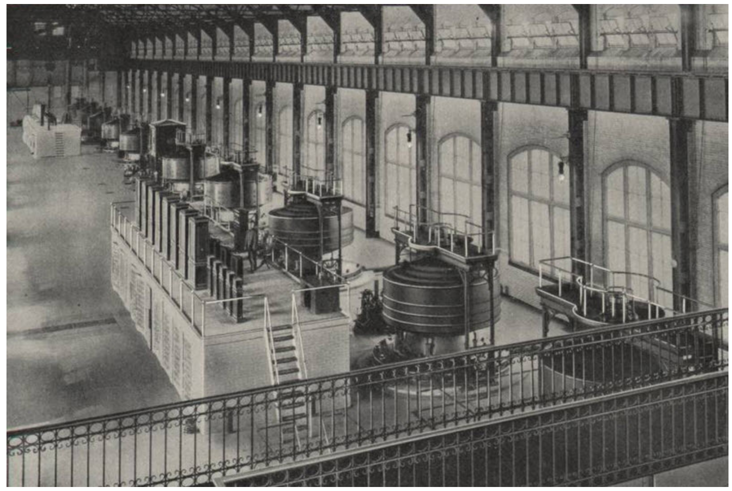


Figure 2: Power station at Niagara Falls
(Image credit: Wikipedia)

But the problem with DC was transmission. Edison, when asked by cable--he was in Europe at the time--about the prospect of transmitting large-scale power from Niagara Falls to Buffalo, wired back: "No difficulty transferring unlimited power. Will assist." But indeed there was difficulty, as Edison well knew. And neither he nor anyone else figured out how to resolve it using DC. The DC transmission problem was fundamental. Based on Ohm's Law, efficient and economical transmission requires high voltage (raising the voltage causes increased flow of current, while the resistance remains constant, thus lowering the resistance per unit flow of current). Too high a voltage for practical uses, such as the operation of lights or motors.

AC, on the other hand, had the transformer for raising or lowering voltage. The transformer was based on phenomena discovered by Danish physicist and chemist Hans Christian Oersted (1777-1851) and English scientist Michael Faraday (1791-1867). Oersted found that an electrical current produces a magnetic field around it. Faraday found that a conductor (wire) cutting through a magnetic field creates a current in the wire. As a result, an alternating current in a (primary) conductor, because of the constantly changing direction of the current, and thus constantly changing direction of the magnetic field, will induce a similar current in a nearby (secondary) conductor. In the transformer, the conductor wires are formed into coils to enhance the magnetic field and induction effects, and by varying the ratio of turns in the primary and secondary coils, the transformer can be used to change the voltage in the secondary. An effective transformer was developed in 1886 by William Stanley, then working for Westinghouse.

But the transformer phenomenon doesn't work with DC because in DC the direction of the current--and thus the direction of the resultant magnetic field--doesn't change. So that for a second conductor to continuously cut across the magnetic field, the conductor would have to be made to move back and forth across the field. As a result, DC voltage cannot easily be manipulated, and so DC is not readily transmissible. In fact, the service areas of the DC central stations were limited to about a square mile per station.

An additional but related consideration was that the Niagara project--again because of the magnitude of the power that would be produced--would have to be a universal system. That is, from one source it would have to be capable of being used in various ways, at various voltages, as AC and DC, for everything from lights to large and small motors. DC--going back to the problem with raising or lowering the voltage--lacked this flexibility. It couldn't be customized for many different uses.

In this unit, we will study how AC power is generated and transmitted. For the final project, your group will be acting as an electrical engineering team responsible for designing a miniature electrical generator. In short, a generator converts kinetic energy into electrical energy. The big question for this unit is: How exactly do generators work? In particular, we will explore:

- How can magnetic fields be generated without a magnet?
- How can current be generated through a wire without using a battery?
- How does a transformer raise and lower the voltage in an AC circuit?
- How does the electric grid in the US get the electricity from the power plant to our homes?

Once you've studied the physics listed above, then you can design a generator to complete a designated task. Your task is to build a simple generator and also construct a mathematical model to describe the relationship between the parameters of the physical system and the amount of voltage that is produced.

Activity: Power Bills

In this activity, we will calculate your own personal energy usage. If you have access to a utility bill, bring it in to look at during class.

Part 1: How much energy do you use?

1. Use the Kill-A-Watt meter to measure the power output of various devices. Each group should measure at least three different types of devices. We'll share results with the class.

Appliance	Power (Watts)

Appliance	Power (Watts)

2. Look at the list of appliances and their power ratings. Choose appliances from the list or from your measurements. Record the name and power rating below. Estimate the approximate amount of time the appliance is used each day (multiply by 30 days for the hours/month) and calculate the amount of electrical energy each of the appliances uses in a month.

Appliance	Power (Watts)	#hours/ day	#hours/ month	Energy/month (kWh/month)	Cost/month

3. If electricity costs $0.13 per kWh, calculate the cost of running each of the appliances in above for one month.

Part 2: Power Bills

Look at the power bills that have been provided to you. Each person should have a different bill. Examine it on your own, then compare with your partners to answer the following questions.

1. What is the electrical energy (in kWh) used each month?

2. What is the cost per kWh? (Is the cost per kWh the same on all bills or are there differences?)

3. What are some of the other costs listed on the bill?

4. What factors might account for the differences among electric bills within your group? Identify as many factors as you can and explain what you think is the effect of each factor on the bill.

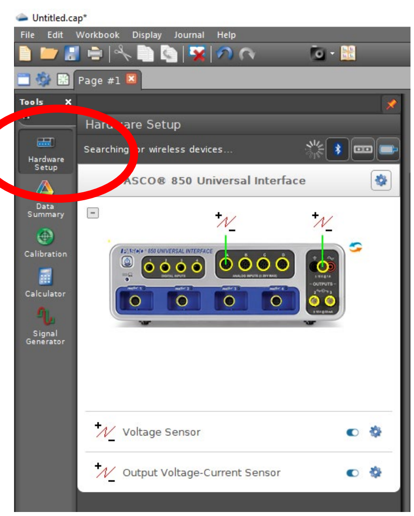
Lab: Generator Model

The goal of this laboratory exercise is to begin to explore how electrical generators work. To do this, we will measure the voltage generated by a miniature generator that you will construct. We'll collect data that can be used to build an empirical model for power generation. This empirical model will then be used to motivate a more detailed study of electricity and magnetism.

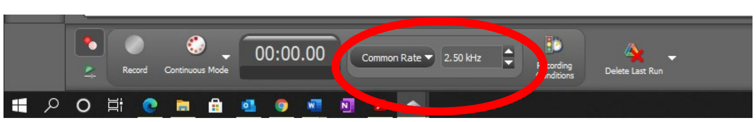
Part 1: Build a Generator

1. Follow the instructions provided to build the simpleGEN generator. Materials are provided so that you can build two generators per group.
   - You will need at least 100 turns of coil to light up the LED.
   - To start, build your two generators so that they have different numbers of coils, and then you can change the other parameters later.
2. Connect each of your generators to the LED bulbs. Spin the magnet and see if the LED lights up. If not, you may need to modify your generator in some way. (The instructions sheet has a page on troubleshooting.)
   - If you got the bulb to light, did it flicker? Why?
   - Was the generator harder to spin with the LED attached?

Part 2: Measure the Voltage Output



1. Connect your generator to the Pasco Universal Interface box to measure the Voltage output. You will need to click on the "Hardware Setup" button to add the voltage sensor.
2. Use the menu at right to add a graph of Voltage vs. Time to the display by dragging the icon into the workspace.
3. At the bottom of the screen, change the collection rate (Common Rate) to 500 Hz.



4. Spin the magnets and confirm that some voltage is being produced. What does the graph look like? Sketch the graph here:

5. Use the curve fit feature to fit the voltage output to a sine wave. From this fit, you can read off the angular frequency ($\omega = 2\pi f$) and maximum voltage.
6. Play around with your group's generators. You can use the drill to control the rotation speed. Record at least three observations:

7. Perform a series of experiments to answer the following questions:
   - How does the number of turns in the coil affect the maximum output voltage?
   - How does the speed of the magnet rotating affect the maximum output voltage?
   - How does the strength of the magnetic field affect the maximum output voltage?

Collect quantitative data and make graphs to support your claims. Be specific as possible. Fill in the table below to organize your thoughts. Be prepared to share your results with the class.

Claim	**Evidence**	**Reasoning**

Part 3: Developing a Model

Our goal here is to develop an empirical model based on the data we collected. We will then compare the empirical model to a theoretical model based on Faraday's Law of Induction.

1. You investigated three questions above. Write a mathematical expression that relates the maximum voltage to each of those parameters. (Note that the ∝ symbol means "proportional to".)

   $V_{max} \propto$

2. We also saw on the graphs that the output voltage is not constant, but has some time dependence. What is the function that describes this time dependence?

   $V(t) \propto$

3. Combine your expression for maximum voltage with the time-dependent expression to find an empirical model for generated voltage. Why can we only say "proportional to" and not "equal to"?

   $V(t) \propto$

4. Why do you think the electricity generated by this type of generator is called an alternating current (AC)?

5. Based on what you've observed in this activity, what are the key components of an electric generator?

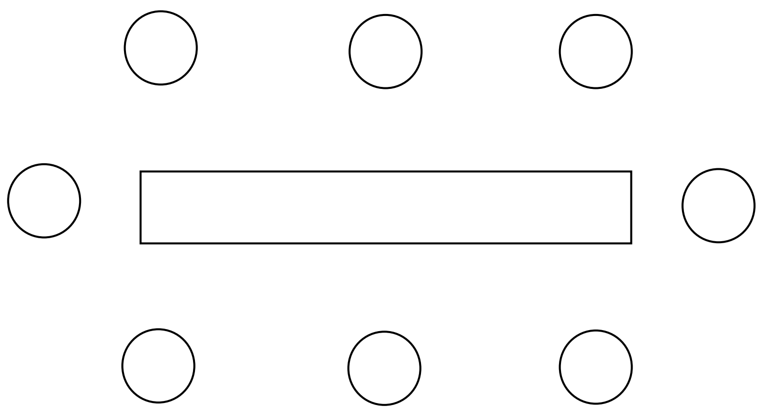
6. What questions do you have about the interactions among these components? Write at least three.

Activity: Electromagnetism Investigations

The relationship between electricity and magnetism was a hot topic of study in the late 1800's. Although magnets found in nature had been known since ancient times, the invention of the battery gave scientists greater control over electricity, which opened up the door to study electromagnetism.

Part 1: Magnetic Fields

1. Play around with the magnets. Record your observations below:
    a. North/North interactions:
    b. South/South interactions:
    c. North/South interactions:
2. Fill in each circle to indicate the direction of the red tip of the compass needle as the compass is moved around the magnet.



3. Physicists use the conceptual model of a magnetic field to explain observations of how magnets behave. What evidence can you give based on your observations of a magnetic field? How do you know if an object has a magnetic field?
4. Based on your observations, what is the magnetic field representing? Describe in words (not equations, yet!).

Part 2: Electric Fields

Physicists also use electric fields to explain the behavior of electrically charged particles. We'll start by investigating electrostatics. Electrostatics, or static electricity, refers to charges being transfer through a mechanical interaction, like rubbing two objects together.

1. Think back to experiences that you have had with static electricity, such as taking off your hat in the winter, sliding down a plastic slide at the park, or shuffling your feet across a rug and touching your little brother. What evidence do you have that a mechanical force is creating an electric charge on an object?

2. Blow up a balloon and rub it vigorously against the fur or your head. What happens to your hair (or the fur) when you rubbed the balloon on it?

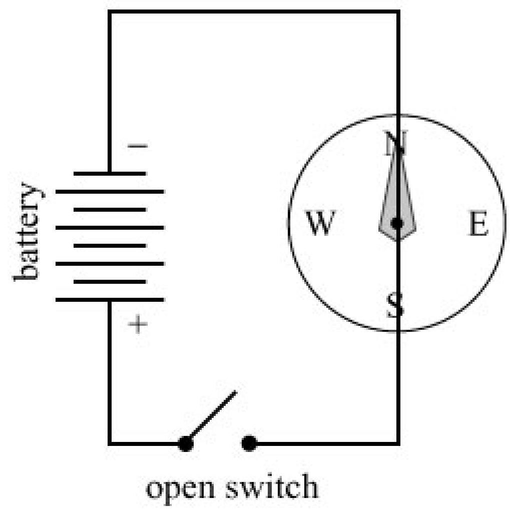
3. Blow up a second balloon and tie the two balloons together with string. Hold them up by the middle of the string while your partner rubs them vigorously against the fur. Record your observations of the balloons.

4. Rip up some paper into small pieces (you won't need too much).Take a plastic rod and rub it on a piece of wool or fur. Hold it near the paper bits. Record your observations.

5. What evidence do you see here for an electric field? What does the electric field represent?

6. Static electricity moves electric charges by mechanical forces, such as friction. Can you think of another way that charges can be moved around?

Part 3: Current-carrying wires



A simple circuit is set up with three batteries and one bulb. The wire should be taped over a compass so that the wire is aligned with the compass direction, as shown in the diagram at right.

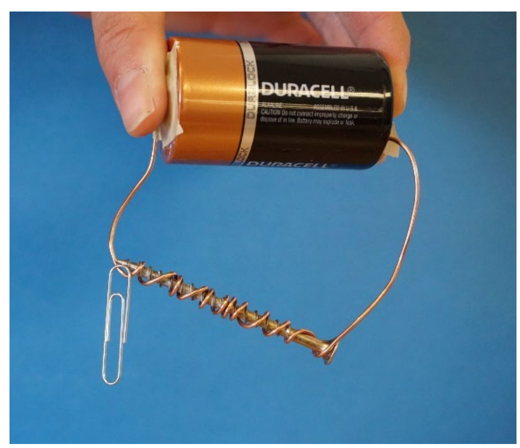
1. Close the switch to connect the battery while watching the compass. What do you observe?

2. Open the switch to disconnect the battery. What do you observe?

3. Switch the wires connected to the battery so that the current flows in the opposite direction. Repeat the experiment. What is different about the result?

4. Explain your observations in terms of the magnetic field. What is causing the compass to move?

On your lab bench, you should have an electromagnet (a large nail with wire wrapped around it). Connect the electromagnet to the battery using alligator clips. Test your device by trying to pick up paper clips. **Be careful not to burn yourself. The batteries get hot!**



5. How many paper clips could you pick up? How could you improve your design to pick up more?

6. What evidence do you have that there is a magnetic field in this system?

7. What factors influence the strength of the magnetic field? You can experiment with the materials on your table to answer this question.

The above observations were first made by Has Christian Oersted in 1820. This was the first discovery of a relationship between electricity and magnetism. Previously, electricity and magnetism were considered to be two separate and distinct areas of physics. The concept of electric and magnetic fields was introduced later by Michael Faraday as a mechanism to explain these observations.

Part 4: Magnets and Coils

Oersted had observed an electric current generating a magnetic field. Faraday wondered if the reverse was true – could a magnet be used to generate an electric current?

1. Move the magnet through the coil of wire. Observe the galvanometer as you do this. A galvanometer is an instrument that can measure small amounts of current. What do you observe as the magnet drops through the coil? What does this tell us about what is happening in the wires?

2. What happens if you hold the magnet steady inside the coil?

3. What factors determine the magnitude and direction of the current generated through the galvanometer? You can use the equipment on your table to explore this question.

4. Now shake the "forever flashlight." What evidence do we have that current is flowing through the wires?

Part 5: Building a Model for Electromagnetism

1. Name two ways that you can generate a magnetic field.

2. Name two ways that you can generate an electric field.

3. Based on what you've learned here, explain Oersted's observations in terms of the electric and magnetic fields.

4. Based on what you've learned here, explain Faraday's observations in terms of the electric and magnetic fields.

5. Based on what you've learned here, explain how a generator works (qualitatively) in terms of the electric and magnetic fields.

Reading: Electric Forces and Fields

Generating electricity requires that electrons be moved through wires in the form of electric current. A generator is a device that causes the electrons to move. But before we can understand exactly how the generator moves electrons, we need to learn some basics of electric forces and fields.

The study of electricity can be broken down into two parts: (1) electrostatics and (2) current electricity. The difference is in what causes the charges to move. In electrostatics, electric charges are static (that is, not moving) unless a mechanical force, such as friction, is applied. In current electricity, we study the effects of many charges moving through circuits in the form of a current. The properties of the charges are the same in both cases, but the observable phenomena are very different.

Properties of Electric Charges

There are two types of charge: **positive** and **negative**. Without an electrometer, it is impossible to tell if something is positively or negatively charged; the observable effects are the same. We can observe that:

(1) Like charges repel

(2) Opposite charges attract

These observations tell us that charges are somehow able to affect each other over a distance. In other words, the charged objects do not have to touch in order to cause a change in the system. (More on the electrical force below.) Charge is measures in units called **Coulombs** (C). One Coulomb is a lot of charge! Typically, we will see values of microCoulombs (1 μC = 10^{-6} C) or nanoCoulombs (1 nC = 10^{-9} C).

What is difficult to observe is that **charge is quantized**. Our current understanding of the structure of the atom tells us that there is a positively charged nucleus surrounded by a cloud of electrons. The electrons are not held as tightly as the protons and neutrons in the nucleus; they can be easily moved from object to object. As a consequence, objects become charged if they have too many or too few electrons.

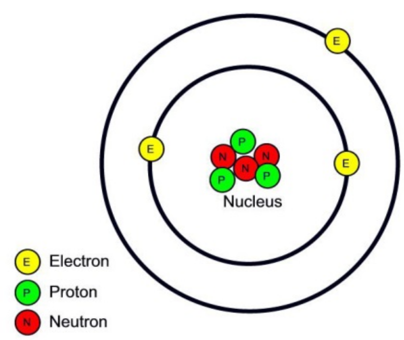


Figure 1: Bohr model of an atom. Note the electrons are not held tightly and are easy to remove.

Millikan's famous oil drop experiment determined that the charge on an electron is 1.6×10^{-19} C. This is the **fundamental unit of electric charge**, which we abbreviate with a little *e*. You will often be given the charge on an object as a multiple of the electron's charge (e.g., a carbon nucleus has a charge of +12*e*). Charge quantization means that we can only ever have an object charged with an integral number of electrons. You can't have half of an electron on an object.

Insulators and Conductors

The properties of a material determine how easy it is for electrons to move around in a given object. In conductors, which are usually metals, atoms are closely packed together and electrons can easily move around because the electron cloud is shared among all of the atoms (see Figure 2). On the other hand, insulators can be materials like glass or plastic with complicated molecular structures, which make it much more difficult for the electrons to move around freely. As a consequence, charge can be transferred to or from an insulator using a mechanical force, such as friction. Once the charge is transferred, the charge will stay where you put it.

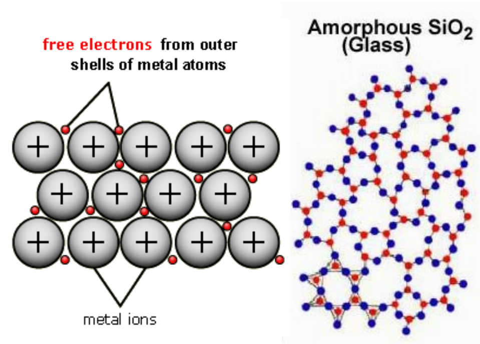


Figure 2: Molecular structure of a conductor. Molecular structure of an insulator (right).

If a conductor has free electrons (as most do) they will be evenly distributed on the surface of the object, along with positive charges, in such a way that the overall charge on the object is neutral. By rubbing two such materials together it is possible to mechanically redistribute the electrons so they are temporarily unevenly distributed. When this happens, there is a slight <u>negative</u> charge where the electrons are concentrated and a slight <u>positive</u> charge where there is an absence of electrons.

Charge Transfer

There are two ways to transfer charge to another an object: (1) charging by conduction and (2) charging by induction.

Charge is transferred by **conduction** when two objects physically touch. Conduction is most effective if the objects are conductors because it is easier for the electrons to redistribute themselves, but it can also happen in insulators as well.

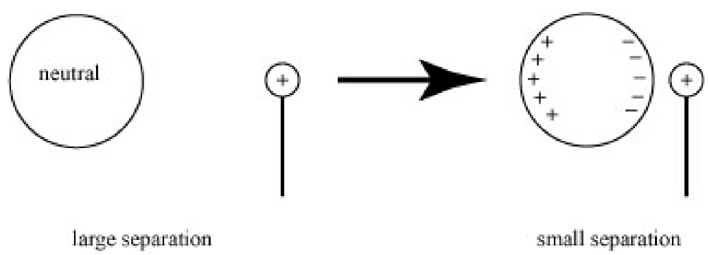


Figure 3: Diagram of inducted charge on a conductor.

Charging by **induction** occurs when a charged object is brought near an uncharged object, but they do not touch (see figure). The charged object may attract or repel electrons in the neutral object, which creates a net charge at one location or another. This is called an **induced charge**, an uneven distribution of charge in an overall neutral object.

Grounding

The earth is considered to be an infinite source or sink for electrons. If you touch a charged object to ground (either the actual earth or a wire that is connected to the ground), it will gain or lose the electrons necessary to bring the object back to neutral. By touching an object with your hand, you are grounding it because you are standing on the ground.

Electric Force & Electric Field

Now we know something about charges, but one burning question still remains: **Why do the charges move?** You know that you can get current to flow through a circuit by connecting it to a battery or other voltage source. Here we will show how voltage is related to force and energy.

We need to think about why anything moves. Recall Newton's 2nd Law: $\sum \vec{F} = m\vec{a}$. If we want the electrons to move through the circuit, we need to apply a force. The force that makes the electrons move is called the **Coulomb (or Electrostatic) Force**. This is what we call a "field force" because the charges do not have to come in physical contact to feel the force.

The force felt by the charge is equal to the charge times something called the **Electric Field**:

$$\overrightarrow{F_e} = q\vec{E} \qquad (1)$$

Where F_e is the electric force measured in Newtons, q is the charge measured in Coulombs, and E is the electric field measured in Newtons per Coulomb (N/C). This relationship tells us that as long as there is an electric field, there will be an electrostatic force. The question is then, **how do we get an electric field**? To answer this, think about the situations in which we see charges moving. In electrostatics, charged objects attract or repel each other, which means there is an electric force, and thus charged objects must have an E-field. In circuits, the voltage from the battery creates a current in a circuit. We can think of this as a force acting on a series of charges, which means the battery creates an E-field in the wires. Unlike a force, the electric field exists even if it is not interacting with anything else. It is a property of a system.

Now let's work through the math to see how exactly voltage is related to electric field. In our study of circuits, we defined voltage or **potential difference** to be energy per charge:

$$\Delta V = \frac{energy}{charge} = \frac{\Delta U}{q} \qquad (2)$$

Where ΔV is the potential difference, ΔU is the change in electric potential energy, and q is again the charge. We also know that work is defined as a change in potential energy, which can be related to voltage:

$$W = -\Delta U$$

$$W = -q\Delta V$$

$$\Delta V = \frac{-W}{q} \qquad (3)$$

We also can define work as a force applied over a distance:

$$W = F_x \Delta x \qquad (4)$$

Where W is the work measured in Joules (1 J = 1 Nm), F_x is the force in the x-direction, and Δx is the distance over which the force is applied. If the force applied is an electric force, then we can plug equation (1) into (4) and get the work done by the electric field to be:

$$W = q\vec{E}\Delta x \qquad (5)$$

By combining equations (3) and (4), we get a relationship between electric field and potential differences:

$$\Delta V = \frac{-W}{q} = -\frac{qE\Delta x}{q} = -E\Delta x$$

$$E = -\frac{\Delta V}{\Delta x} \qquad (6)$$

The equation above shows that mathematically, *electric field is the slope of the voltage over distance*. (It also gives us a new unit for electric field: Volts per meter, which is equivalent to Newtons per Coulomb. You will see them used interchangeably.) Generally, the potential difference (or voltage) changes over a given distance. For example, as you get farther from a positively charged object, the voltage decreases.

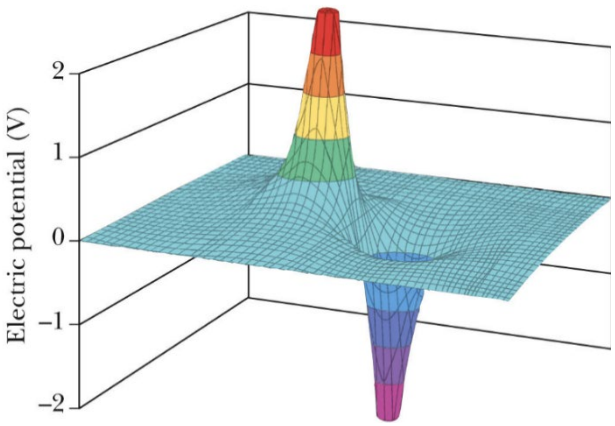


Figure 4: Electric potential map of a dipole (one positive point charge(red) and one negative point charge(purple)).

It is helpful to visualize the potential as a topographic map, as shown in Figure 4. The positive charge is at the top of the hill (shown in red). The potential drops off as you get farther away from the charge, as if you were going down a hill. In this case the E-field would point away from the positive charge. If you think of the voltage as a hill, the electric field always "points downhill".

The negative charge is down in the valley (shown in purple). The potential increases as we move away from the negative charge, as if we were climbing uphill. In this case, the E-field points away from the positive charge and towards the negative charge.

Figure 5 shows a positive point charge. The E-field lines are shown in orange, pointing radially away from the charge. This indicates the potential is decreasing as you get farther away from the charge. The blue circles are called **equipotential** lines, lines of equal potential. In other words, these indicate places where the voltage (potential) does not change (is equal). *No work must be done to move a charge along those lines.* It does take work (or energy) to cross lines.

These circles are analogous to the elevation on a topographic map. Each line is draw at a given voltage interval (for example, every 3 Volts) just like in a topographic map a line would be drawn for every so many feet. The closer the lines are together, the steeper the slope. We can see in Figure 5, the slope is steeper near the charged particle. Equation (6) tells us that the electric field is stronger closer to the charged particle.

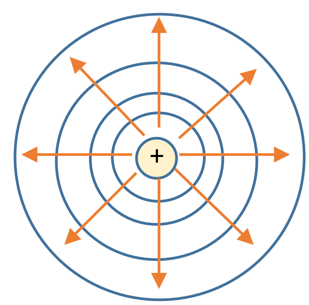


Figure 5: Electric field map of a positive point charge. The equipotential lines are shown in blue.

Reading: Magnetic Forces & Fields

Magnetic Fields

A magnetic field is a conceptual tool that physicists use to explain the behavior of magnets. As a child, you likely discovered that magnets have two ends – the north pole and the south pole. Magnets with like poles facing each other repel, while opposite poles attract. This attraction or repulsion can be thought of as a force, an interaction between two objects. The limitation with using forces to describe magnets is that they are dependent on interactions. Is a magnet still a magnet if there isn't an interaction with another object? What if there isn't a second object to interact with? (If a tree falls in the woods and no one is around, does it make a sound?)

Generally speaking, a field is a property of a system, independent of any interactions. For example, the Earth's gravity can be thought of as a field. The gravitational field exists, even if it isn't interacting with another object. When we do drop an object, then that object feels a gravitational force. Magnetic fields are similar. A permanent magnet always has a magnetic field, regardless of whether or not it is attracting or repelling another object. (There are also ferromagnetic materials, which are only magnetic in the presence of an external magnetic field.)

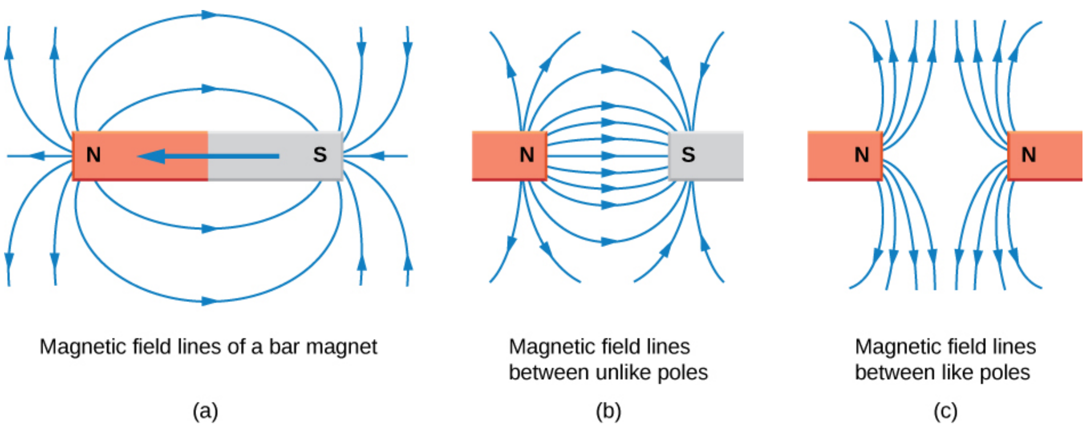


Figure 1: Magnetic Field Lines (Image credit: Physics LibreTexts)

Magnetic fields are represented as $\vec{B}$ and measured in units of Teslas (T). One Tesla is a large magnetic field. The Earth's magnetic field is 25 to 65 microTeslas ($1\ \mu\text{T} = 10^{-6}\ \text{T}$). Magnetic fields (like electric fields) are vector quantities, so we always will report both a magnitude and a direction.

Figure 1 shows several ways of arrangements of magnetic fields around bar magnets. The bar magnet represents a permanent magnet, typically made of an iron alloy. For a permanent magnet, the field is caused by the alignment of the atoms in the molecular structure. (This is why magnets can become demagnetized when you drop them; it physically changes the alignment of the atoms.) The Earth also has a magnetic field, which is generated by molten iron in the core. The Earth's magnetic field is what makes a compass needle point towards North.

Early Studies in Electromagnetism

In 1820, Hans Christian Oersted first observed that a compass was affected by current running through a wire. Taking the compass to be evidence of a magnetic field, he concluded that the current in the wire was generating a magnetic field (see Figure 2). This result was puzzling to scientists because at the time, they had only experienced forces running through wires (i.e. electrical forces). How could the moving charges be generating a force that was outside the wire?

Observations of the magnetism around current-carrying wires were made by many scientists, but it took a while for the idea of magnetic fields to emerge. André Marie Ampere continued Oersted's work by placing two current-carrying wires near each other and measuring the force of the interactions. It wasn't until the 1840's when Michael Faraday first came up with the idea of electric and magnetic fields as way to explain his observations. (We'll learn more about Faraday in the next reading.) This idea was so abstract that it was not readily accepted by scientists.

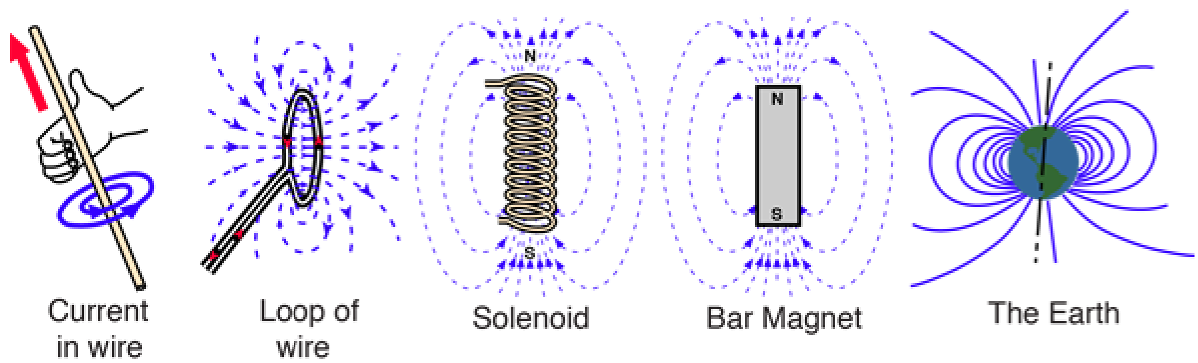


Figure 2: Magnetic Field Sources (Image credit: [Hyperphysics])

Finally, in 1861, James Clerk Maxwell published a paper that laid the groundwork for all of electromagnetism. In his paper, he developed a mathematical model for electric and magnetic fields. Maxwell's equations proved to be enormously useful in explaining all electromagnetic phenomena and are still used today. In honor of the scientists who had worked on these problems before him, he named the equations after them: Gauss's Law; Faraday's Law; Ampere's Law; and the Lorenz Force Law.

Ampere's Law

Ampere's Law describes the strength of the magnetic field around a current carrying wire. Key to understanding his law is to know that the magnetic field forms circles around the wire, as shown in Figure 3. The direction of the magnetic field is determined by the direction the current is flowing through the wire. Because we now are working in three dimensions, we need to introduce a new notation for vectors:

X indicates that the vector (current or force or field) is <u>into</u> the page. Imagine that an arrow is pointing towards something far away. All that you would see are the feathers on the back end, which make the shape of a cross.

• Indicates the vector is coming <u>out</u> from the page. This is the tip of the arrow pointing at you.

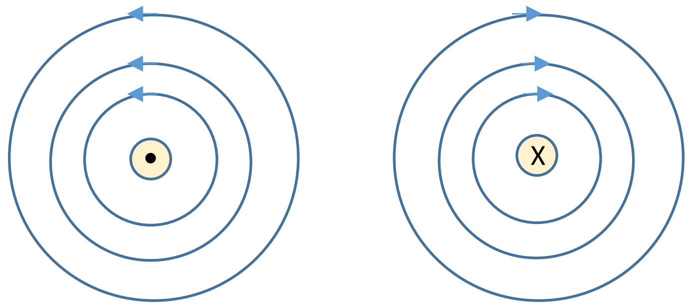


Figure 3: Magnetic field lines around current-carrying wires.

For a wire that has the current coming out of the page (as shown on the left above), the magnetic field lines will be directed counter-clockwise. For the wire with the current traveling into the page, the magnetic field lines are directed clockwise. One way to remember this is called the **Right Hand Rule**. If you put the thumb of your right hand in the direction of the current, imagine you are grabbing the wire. Your fingers will curl in the direction of the magnetic field (Figure 2).

The magnitude of the magnetic field around a single straight wire is given by the following expression:

$$B = \frac{\mu_o I}{2\pi r} \qquad (1)$$

Where B is the magnitude of the magnetic field measured in Telsas; μ_o is a constant called the permeability of free space ($\mu_o = 4\pi \times 10^{-7}\ \mathrm{T/m}$); I is the current in Amps; and r is the radius of the circle around the wire, or in other words the distance from the wire to the point where you want to find the magnetic field. One important thing to note is that the strength of the field drops off as $1/r$, which means the strength decreases as you get farther from the wire.

If the wire is curled into a circular shape, we can still apply Ampere's Law. The magnetic field lines now curve around the loop of wire and begin to resemble the field lines from a permanent magnet, as shown in Figure 2. An extended coil of wires is called a **solenoid**. Usually when we talk about an electromagnet, we are referring to a solenoid. A solenoid is useful in many scientific experiments because it creates a uniform magnetic field inside the coil. The magnetic field generated by a solenoid can also be found using Ampere's Law:

$$B = n\mu_o I \qquad (2)$$

Where n is the number of turns per unit length ($n = N/L$), and the other parameters are the same as described above. Note that the magnetic field of the solenoid is independent of the radius of the coil. This expression gives the strength of the magnetic field within the coil, and does not describe the magnetic field at the ends or outside of the solenoid.

Magnetic Forces on Charged Particles

Ampere's Law shows us that a stream of charged particles can generate a magnetic field. This indicates that there is a fundamental relationship between charged particles and magnetic fields. In particular, there is a magnetic force that acts on charged particles moving through an external magnetic field. This is called the **Lorentz Force Law**, another mathematical model perfected by Maxwell. Let's write it down, and then we'll dissect it.

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad (3)$$

In this equation, $\vec{F}_B$ represents the magnetic force, q represents the charge, $\vec{v}$ is the velocity of the charged particle, and $\vec{B}$ is the magnetic field. You may be wondering what the $\times$ means in this equation. This is a mathematical symbol for the **vector cross product**. When you multiply vectors together, there are two ways to do it:(1) You can multiply together the parallel components, which is called the dot product (and represented by a big dot). The dot product is sometimes called the scalar product because the result is a scalar quantity (one that has no direction). (2) You can multiply together the perpendicular components. The cross product, as shown in the equation above gives a result that is a vector. This means we need to worry about both the magnitude and direction of the resulting force.

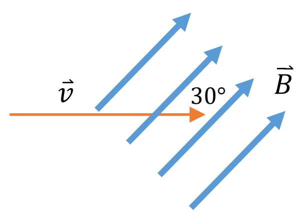
Let's look at the magnitude first. We can rewrite the expression in terms of the perpendicular component:

$$\left|\vec{F}_B\right| = qvB_{\perp}$$

This tells us the *maximum force* felt by the particle occurs when the velocity is perpendicular to the magnetic field: $\left|\vec{F}_B\right| = qvB$

If the velocity is in the same direction as the magnetic field, there is no perpendicular component. In this case, there is *no force* felt by the particle: $\left|\vec{F}_B\right| = 0$

If the velocity is at an angle with the magnetic field, then *only the perpendicular component* of the field will contribute to the force. For example, look at the situation below:



In this case, the direction of the velocity of the particle is at a 30° angle with the direction of the magnetic field. To find the perpendicular component of the magnetic field, we need to revisit some trigonometry. The perpendicular component of the magnetic field would be oriented vertically, which would be the sine of the angle:

$$\sin\theta = \frac{B_{\perp}}{B}$$

$$B_{\perp} = B\sin\theta$$

The cross product also tells us the direction of the force. It is always oriented perpendicular to both the magnetic field and the velocity. In the example above, that would mean that it must be either into the paper or out of the paper. To figure out which it is, we use another **Right Hand Rule** (RHR). The RHR is difficult to explain on paper, so it is not a bad idea to look up some videos online to watch someone doing it. You can follow along with the video and you should definitely practice with your actual hands as much as possible.

The Right Hand Rule assigns role to your fingers and thumb. Start by making your hand look like a corner, as shown in Figure 4. The thumb represents the direction of the force; the pointer finger represents the direction of the velocity; and the middle finger represents the direction of the magnetic field. You can then rotate your hand around until it matches the orientation of the problem.

For the example above, orient your hand so that the pointer finger points to the right and your middle finger is toward the top of the page. Your thumb tells you that the force is coming out of the paper. Note that sometimes it is easier to move your paper than your hand!

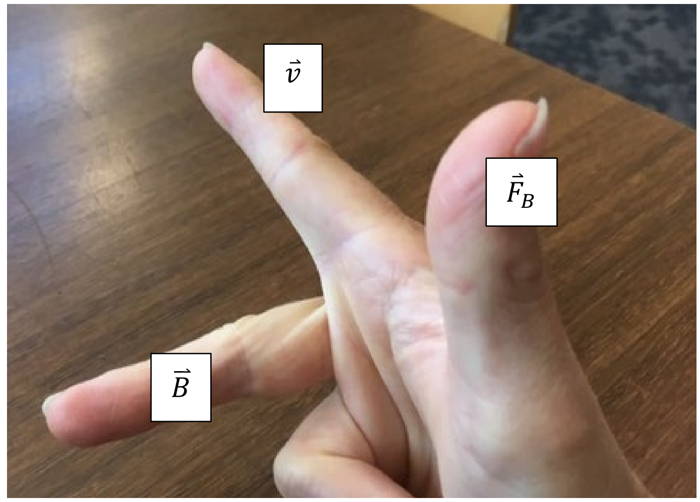


Figure 4: Orientation of your hand for the Right Hand Rule

Lab: Faraday's Law of Induction

Faraday's Law of Induction is named for Michael Faraday, who invented the electric motor and the electric generator. Both of these devices rely on the interactions of electricity and magnetism. Faraday developed the concept of electric and magnetic fields to describe his observations. The mathematical model describing these interactions was later formalized by Maxwell. We'll look at both these representations here.

Part 1: Generate some voltage!

1. Think back to the generator lab. Describe what was physically happening in the system to produce electricity.

2. A key part of Faraday's Law is that a voltage is produced when something is changing. What is changing in the generator?

We'll explore some of these changes in this activity and use the result to derive Faraday's Law of Induction.

3. To begin, connect a coil of wires to the Pasco Universal Interface using a Voltage Probe.
4. Make a graph on the screen that shows voltage as a function of time. Change the sample collection rate (Common Rate) to 100 Hz. The control for this is at the very bottom of the screen, next to where is says Recording Conditions.
5. Hold the coil so it is a few inches above the table.
6. Hit record and drop a magnet through the coil. (Try to catch it in your hand or let it fall on something soft.) What do you observe on the graph?

7. How does the strength of the magnetic field <u>in the coil</u> change as the magnet is dropped?

8. Sit the coil on the table. Repeat the experiment, but do not drop the magnet. Slowly lower the magnet into the center of the coil and let it rest on the table. Slowly raise the magnet out of the coil and observe the voltage that is generated. How is the result different from when you dropped the magnet? Try raising and lowering the magnet at various speeds.

9. Make an argument: does the voltage generated depend on the strength of the magnet or the rate that the magnetic field is changing? Defend your argument with evidence from your experiment.

10. Repeat the experiment using a solenoid with a different number of coils. Do you see any change in the induced voltage? Try at least three different solenoids.

Part 2: Magnetic Flux

Physicists define a quantity called **magnetic flux** to quantify the amount of magnetic field in a given area, such as the coil. Mathematically, flux is defined as:

$$\Phi_B = \vec{B} \cdot \vec{A} \qquad (1)$$

Where Φ_B is the magnetic flux measured in Tesla-meters squared (Tm^2), $\vec{B}$ is the strength of the magnetic field, and $\vec{A}$ is the area of the loop.

The dot in the middle indicates that this is a dot product (or scalar product). Functionally, this means that the magnetic field must be parallel to the area vector. Physicists do something that might seem strange here and define the area vector to be perpendicular to the actual area, as shown in the figure below (left).

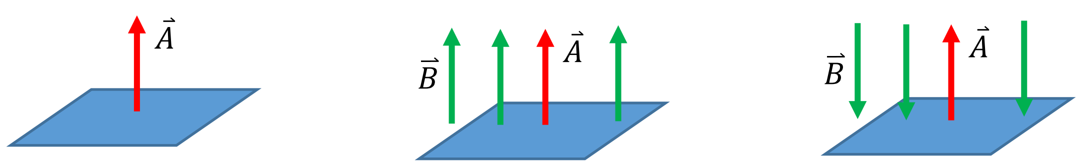


Figure 1: The area vector is always drawn perpendicular to the area (left). The flux is the number of magnetic field lines in a given area (middle and right).

Figure 1 (middle) shows magnetic field lines directed through the area, and in the same direction of the area vector. This is considered to be a positive flux ($+\Phi_B$). The figure on the right shows the magnetic field lines pointed in a direction opposite the area vector (anti-parallel). This is considered to be a negative flux ($-\Phi_B$).

However, our observations tell us that voltage is generated when something is changing. So it is not the absolute flux that is important for Faraday's Law, but the *change* in the flux. If the magnetic field is decreasing in strength (as shown in Figure 2), the flux might still be positive, but the change in flux ($\Delta\Phi_B$) is negative.

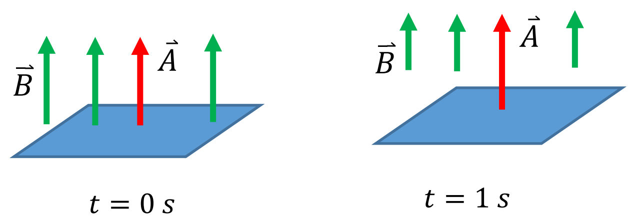


Figure 2: A negative change in magnetic flux due to a decreasing magnetic field.

1. Sketch the magnetic field through the area of the loop at the following times:
    a. Just before you drop the magnet,
    b. As the magnet falls towards the center of the coil,
    c. When the magnet is just above the middle of the coil,
    d. When the magnet is just below the middle of the coil,
    e. As the magnet falls away from the center of the coil,
    f. Just before the magnet hits your hand.

As you make your sketches, note how the strength of the magnetic field changes through the center of the coil.

2. Qualitatively describe the flux at each point. Is it large, small, or zero? Is the flux positive or negative?

3. Is the change in magnetic flux from (a) to (b) positive or negative? (b) to (c)? (c) to (d)? (d) to (e)?

4. Compare the sign of the change in flux to the sign of the voltage generated. (Use the graph on the computer.)

5. Write a mathematical model that explains your observations.

6. Would your mathematical model also work to explain how the generator works? Explain.

Part 3: Forces & Fields

In our loop, a copper wire is full of electrons just waiting to become an electric current. Let's think about what we've observed using the force laws:

$$\vec{F}_B = q\vec{v} \times \vec{B} \tag{2}$$

$$\vec{F}_E = q\vec{E} \tag{3}$$

1. Before we drop the magnet, what is the velocity of the electrons in this copper wire? Will the electron feel a magnetic force under these conditions? Explain.

2. Do the electrons feel an electric force under these conditions? Explain.

3. When we drop the magnet, we measured a voltage in the coil. What does this mean for the electrons in the wire? What does this tell us about the existence of an electric field?

Another way to conceptualized Faraday's Law is to say that a changing magnetic field generates an electric field. Then, the electric field in the wires will cause a current to flow through the wires.

4. According to Ampere's Law, what happens when a current flows through the wires?

5. Is it ever possible to have a magnetic field without an electric field? Is it possible to have an electric field without a magnetic field? Explain.

Reading: Faraday's Law

We have now seen several examples of **Faraday's Law of Induction**. Faraday described induction as the process of a changing magnetic field generating an electric field, which induces a current in a circuit. Maxwell formalized the mathematics in the following way:

$$\varepsilon(t) = -N\frac{\Delta\Phi_B}{\Delta t} \qquad (1)$$

Where $\varepsilon(t)$ represents the voltage generated as a function of time[2] (you could equivalently use ΔV), N is the number of turns in the coil of wire, $\frac{\Delta\Phi_B}{\Delta t}$ is the change in magnetic flux as a function of time. Physicists define a quantity called **magnetic flux** to quantify the amount of magnetic field in a given area, such as the coil. Mathematically, flux is defined as:

$$\Phi_B = \vec{B}\cdot\vec{A} \qquad (2)$$

Where Φ_B is the magnetic flux measured in Tesla-meters squared (Tm^2), $\vec{B}$ is the strength of the magnetic field, and $\vec{A}$ is the area of the loop.

The dot in the middle indicates that this is a dot product (or scalar product). Functionally, this means that the magnetic field must be parallel to the area vector. Physicists define the area vector to be perpendicular to the actual area, as shown Figure 1.

Faraday's Law tells us that a voltage can be generated by changing the magnetic flux. This can happen in a number of ways:

- By changing the strength (magnitude) of the magnetic field,
- By changing the area, or
- By changing the orientation between the magnetic field and the area.

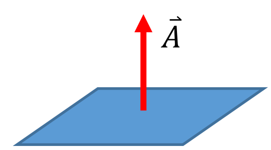


Figure 1: The area vector is always drawn perpendicular to the area

In practice, changing the area does not happen very often, so we'll focus on the other two possibilities. The strength of the magnetic field can be changed by inserting a permanent magnet into a coil of wire. We saw this in one of the earlier activities. In scientific applications, the magnetic field is often changed using an electromagnet. The strength of the electromagnet can be controlled by adjusting the current through the wires.

AC Generator

An **AC generator** converts mechanical energy to electrical energy by spinning one coil of wires inside of another one. One of the coils is an electromagnet, so spinning it creates a changing magnetic field with respect to the other set of coils. Let's look at how Faraday's Law plays out in this case. We'll start by substituting equation (2) into (1) so that we can see the dependence on the magnetic field:

$$\varepsilon(t) = -N\frac{\Delta(BA\cos\theta)}{\Delta t}$$

[2] Scientists still use the old notation of the script epsilon (ε), which was the symbol for electromotive force (emf). This term isn't used anymore because we now think of this quantity as potential difference or voltage, not a force.

Where we have used the definition of the dot product ($\vec{B} \cdot \vec{A} = BA\cos\theta$) to introduce the $\cos\theta$ term. This will allow us to mathematically account for the rotation as a changing angle. The angle between the magnetic field and the coil is changing as a function of time. To see this time dependence explicitly, we can rewrite the angle in terms of the angular frequency:

$$\omega = \frac{\Delta\theta}{\Delta t} \rightarrow \theta = \omega t$$

$$\varepsilon(t) = -N\frac{\Delta(BA\cos\omega t)}{\Delta t}$$

$$\varepsilon(t) = -NBA\frac{\Delta(\cos\omega t)}{\Delta t}$$

Where in the last line, the constant terms have been factored out. This expression is awkward because it has the change of a cosine function. Technically, we need to use calculus to take the derivative of this expression. If you don't know calculus, just close your eyes and the following expression will magically appear:

$$\varepsilon(t) = NBA\omega\sin\omega t \qquad (3)$$

This expression should agree with the empirical model that we found in the generator lab. If I were to hook up the generator to a circuit with some resistance, we could use Ohm's Law to find the current:

$$\varepsilon = IR$$

$$I(t) = \frac{\varepsilon(t)}{R} = \frac{NBA\omega}{R}\sin\omega t \qquad (4)$$

The term alternating current (AC) comes from this expression; the current is oscillating as a sine wave. It is called alternating because the current changes direction – half the time it is positive, meaning it flows in one direction through the circuit, and the other half the time current is negative and flows in the opposite direction.

From equations (3), we can see that the maximum voltage generated depends on the number of turns of wire, the strength of the magnetic field, and the frequency of the rotation:

$$\varepsilon_{max} = NBA\omega \qquad (5)$$

This expression should agree with your results from the Generator Lab. Similarly, we can define the maximum current as:

$$I_{max} = \frac{NBA\omega}{R} \qquad (6)$$

In the US, electrical systems are standardized to 120V AC. (In Europe, the standard is 220V; in Asia it is 240V.) The frequency is 60 Hertz, or 60 cycles per second. (To convert frequency to an angular frequency, recall that $\omega = 2\pi f$.) In the US, we also use something called three-phase (or polyphase) power, which means that our circuits actually consist of three sine waves. All three waves oscillate at the same frequency but are 120^{o} out of phase with each other, as shown Figure 2. This ensures that the voltage generated is consistent. (Polyphase also makes AC motors run smoother because they receive a "kick" three times as often to keep them spinning.)

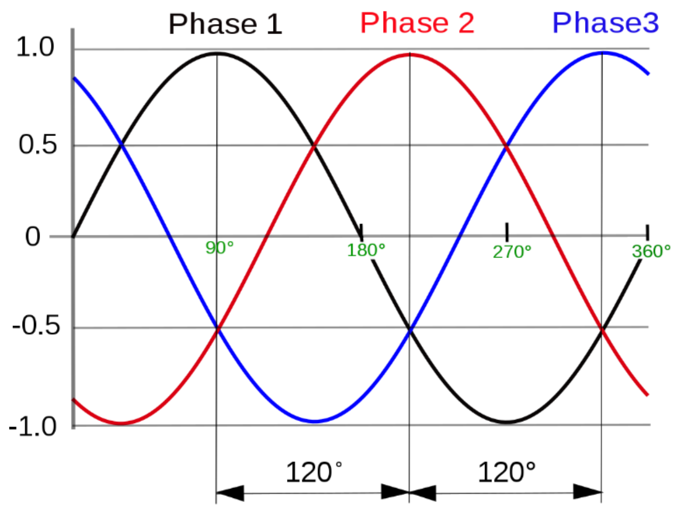


Figure 2: Three phase power. (Image credit: Wikipedia)

AC Motor

An **AC motor** is the opposite of an AC generator; it converts electrical energy into kinetic energy. A motor could be exactly the same device as a generator, just run in reverse. Motors are used in many everyday household appliances: vacuum cleaners, hair dryers, wood chippers, lawn mowers, blenders, food processors, etc.

When the motor begins to run, it acts like a generator and creates a voltage. Does this seem odd? Using a voltage to run the motor generate more voltage? This might seem like a case of runaway energy generation, but before we get too excited, let's look back at Faraday's Law:

$$\varepsilon(t) = -N\frac{\Delta\Phi_B}{\Delta t} \qquad (7)$$

You might not have noticed before, but there is a negative sign in front of the right-hand side. This is the most famous negative sign in all of physics; it even has its own name: **Lenz's Law**.

In the case of the motor, or any coil of wire connected to an electrical circuit, this tells us that the voltage generated will *oppose* the current through the wire. What this means is that the voltage will work against the primary power supply to the device. This is called **back emf** (ε_{back}). We can write this mathematically as part of the Loop Rule (energy conservation) equation for circuits:

$$\varepsilon_{source} - IR - \varepsilon_{back} = 0$$

Let's take an example of a saw blade. A typical table saw can run at speeds up to 50,000 rotations per minute. This is an angular frequency of about 5,000 radians per second. Even with a relatively small coil, this rotation speed can generate a lot of voltage! Let's say the back emf generated by the saw is 70V, then the overall voltage in the circuit would decrease by 70V when it is running. If we are connected to a standard US system, the saw would draw 120V from the wall outlet. However, once the saw blade begins to spin, the overall voltage in the circuit would only be 50V (120V - 70V).

How does this affect the system? A device running at 120V will draw much more current than a device running at 50V. If you have ever noticed the lights dim when you turn on the vacuum, this is why. Before the motor starts spinning, the vacuum draws a lot of current into its branch of the circuit. This takes current away from the lights and other devices that are connected on the same circuit.

Figure 3: A transformer at a substation. (Image credit: [Spencer Municipal Utilities])

Transformers

Transmission lines are wires that have a small amount of resistance, but over many miles this resistance adds up to a significant energy loss. Recall from Ohm's Law that $\boldsymbol{\Delta V = IR}$. If we combine this with the expression for power, we get an equation for the rate that energy is lost in the transmission lines:

$$P_{lost} = I\Delta V = I^2 R$$

This energy lost through resistance is called **Ohmic heating**. You may have noticed this phenomenon when a device, such as your laptop power supply or your electric toothbrush charger, is plugged in for a long time and begins to heat up. Ohmic heating depends on current-squared. This means that one way to reduce energy loss is to reduce the current in the system.[3]

A **transformer** is a device that can change the voltage, and therefore the current, of an AC electrical system. Transformers were the key invention that made the AC power grid possible. Power plants generated electricity at a relatively low voltage, which is then "stepped up" and transmitted to customers over high voltage lines. The voltage is then "stepped down" to a lower voltage in your neighborhood substation.

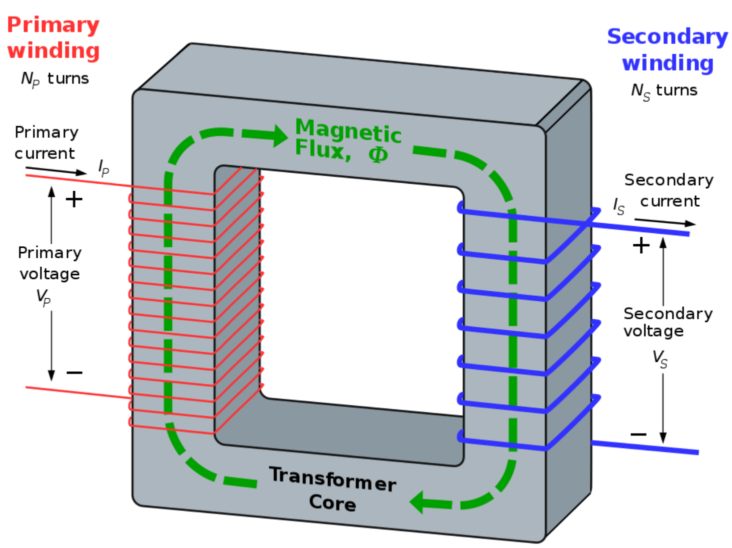


Figure5: Diagram of a transformer. (Image credit: [Wikipedia])

[3] Another way would be to reduce resistance. To reduce the resistance in the wires, they would need to be made of larger diameter wire. Given the price of copper, this is too expensive to be feasible.

A transformer is a simple device. There are two coils of wire that share an iron core. The iron core serves to magnify the magnetic field and increase the flux in the system. The coil connected to the power supply is called the **primary coil** (or primary winding). The coil that has an induced voltage is called the **secondary coil** (or secondary winding). We can apply Faraday's Law to each of the coils in this system:

$$\Delta V_1 = -N_1 \frac{\Delta\Phi_B}{\Delta t} \qquad\qquad \Delta V_2 = -N_2 \frac{\Delta\Phi_B}{\Delta t}$$

Because of the shared iron core, the coils have the same change in flux, which means we can set these two expressions equal:

$$\frac{\Delta V_1}{N_1} = \frac{\Delta V_2}{N_2}$$

A **step-up transformer** will convert a lower voltage to a higher voltage ($\Delta V_2 > \Delta V_1$). In this case, the number of turns in the secondary coil should be larger than the number of turns in the primary coil ($N_2 > N_1$). A **step-down transformer** takes a high voltage and converts it to a lower voltage ($\Delta V_2 < \Delta V_1$), which means that the number of turns in the primary coil is larger than the number of turns in the secondary coil ($N_1 > N_2$).

For an ideal transformer, energy is conserved. This means that rate energy is generated in the primary coil must be the same as in the secondary coil. Mathematically, we express this relationship using power:

$$P_1 = P_2$$

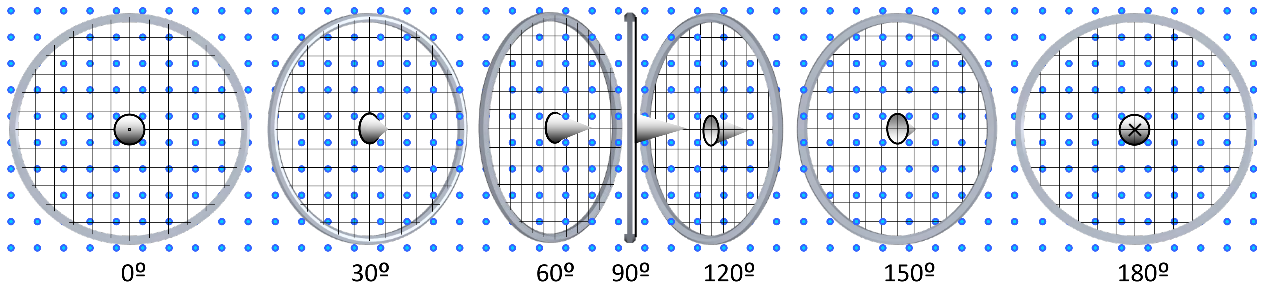
$$I_1 \Delta V_1 = I_2 \Delta V_2$$

This tells us something important about the current in the system. Stepping up the voltage reduces the current in the wires, compared to the current generated at the power plant. This is necessary to reduce the energy losses due to Ohmic heating over large distances. Here we see the main advantage to the AC power grid – The transformer only works for AC electricity. DC power would not have a changing flux, and therefore would not induce a voltage in the secondary coil.

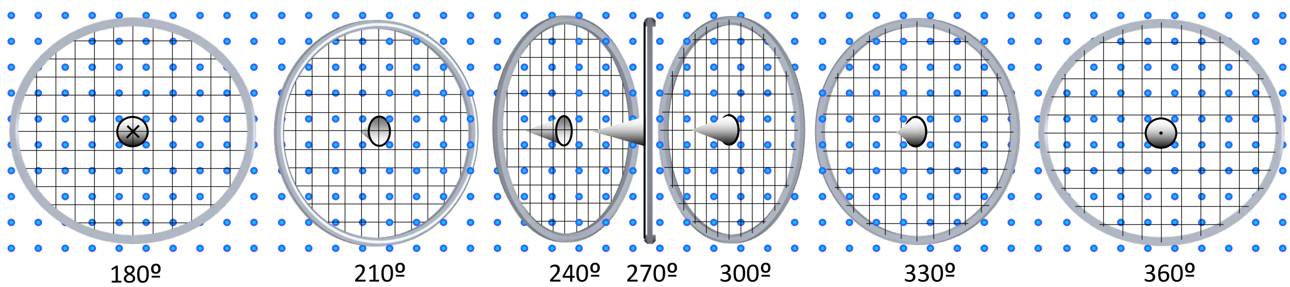
Figure 6: Transformer on a utility pole (left) and at a rural residence (right).
(Image credits: [Wikipedia](#) & [Electrical Engineering Portal](#))

Tutorial: Generator

The generator that we built consists of a uniform magnetic field rotating through a coil of wire. This is equivalent to a fixed magnetic field and a rotating coil. Below are series of snapshots of a circular loop rotating through 360º in a uniform magnetic field.



You can imagine a tennis racket whose area orientation ($\vec{A}$) is defined by the arrow (cone) pointing normal to the surface. The angles shown are between this normal area vector and the $\vec{B}$-field which points <u>out of the page</u> and is represented by the dots. The goal here is to understand conceptually how the magnetic flux through the loop changes as the loop rotates.

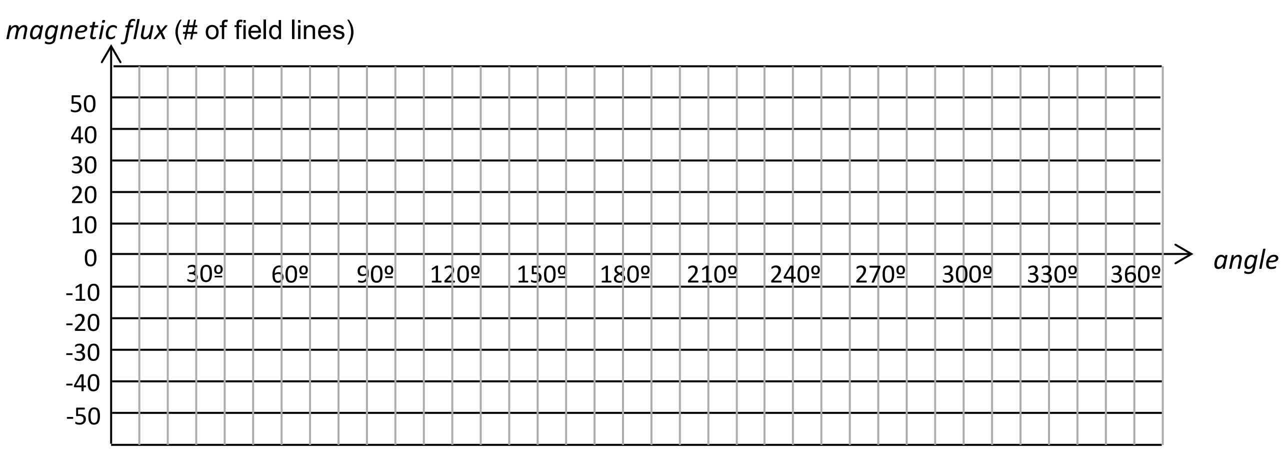


Magnetic flux is defined as the magnetic field in a given area:

$$\Phi_B = \vec{B} \cdot \vec{A}$$

In the case shown here we can estimate the relative flux by just counting the number of field lines that pierce the surface. Note that the diagram shows that no field lines pierce the surface at 90º. **Don't forget that flux can be positive or negative. Include the signs on your graph.**

1. Plot this data on the graph given below.



2. At which angles is the flux zero? At which angles is the flux maximum?

3. At which angles is the flux changing the fastest? At which angles is the flux momentarily not changing?

4. Look at your graph. What mathematical expression can you write to describe the flux as a function of angle? In other words, what mathematical function describes the shape of this graph (e.g., tangent, cosine, exponential, power function)?

   $\Phi_B(\theta) =$

In rotational motion, we define the angular speed or angular frequency in terms of the change in angle:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

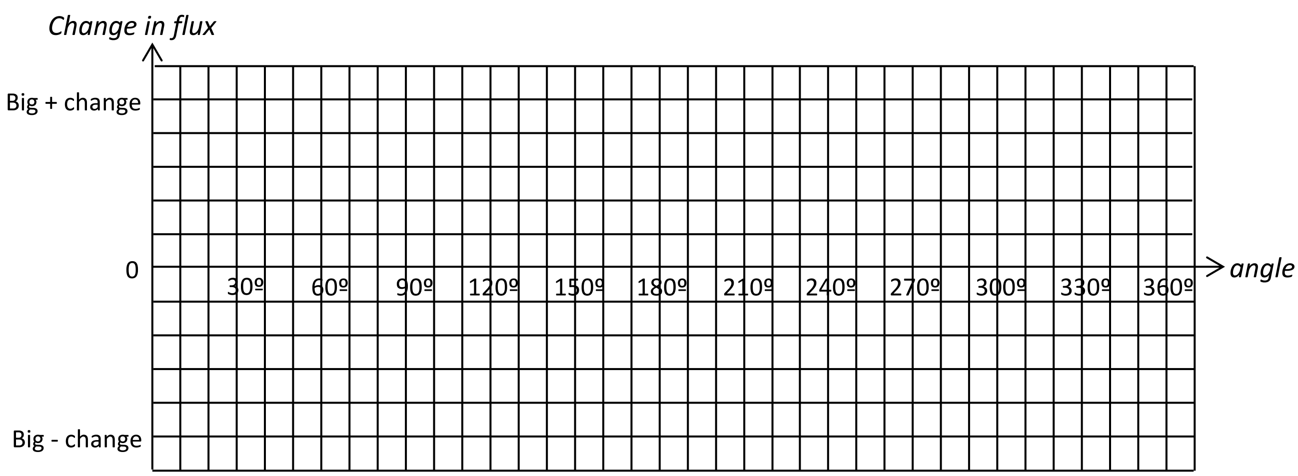
Where ω is the angular frequency, θ is the angle, and t is time. Angular frequency is measured in units of radians per second, which means the angle would be measured in radians (not degrees). We can rewrite the expression above to describe how the angle changes with respect to time:

$$\theta(t) = \omega t$$

5. Rewrite the expression above (#4) in a way that shows how flux depends on time.

   $\Phi_B(t) =$

6. Redraw the graph above, but plot change in flux rather than flux. You can do this qualitatively. Start by marking the points where the flux is changing the most and the least, and then connect the dots in a smooth curve.



7. Look at your graph. Write a mathematical expression that describes the change in flux over time.

$$\frac{\Delta\Phi_{\mathrm{B}}(t)}{\Delta t} \propto$$

Using calculus, we can take the derivative of the flux equation and find that the change in flux depends on time in the following way:

$$\Phi_{\mathrm{B}}(t) = BA\cos\omega t$$

$$\frac{\Delta\Phi_{\mathrm{B}}(t)}{\Delta t} = -BA\omega\sin\omega t$$

Faraday's Law tells us that a changing flux produces a voltage:

$$\Delta V = -N\frac{\Delta\Phi(t)}{\Delta t}$$

$$\Delta V = NBA\omega\sin\omega t$$

This is what we call alternating current (AC) voltage because the voltage, and therefore the current, oscillate sinusoidally in the circuit.

8. Compare this expression to your empirical model from the generator lab. Explain any discrepancies.

9. Explain how you could test this theoretical model. What data would you collect? How would you analyze it?

Based on an activity developed by Larry Watson for the Physics Learning Center at the University of Wisconsin-Madison.

Tutorial: AC Power

Alternating current and voltage are constantly oscillating, which means they are dependent on time. We can write this time dependence out explicitly:

$$I(t) = I_{max} \sin \omega t$$

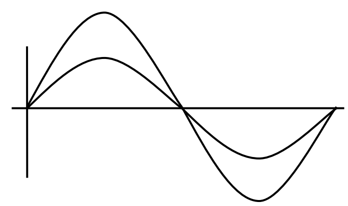
$$\Delta V(t) = \Delta V_{max} \sin \omega t$$

This poses a challenge when we go to calculate the amount of current and voltage in a system. Do we want to know the current and voltage at a particular moment in time? Do we want to know the maximum current or voltage in a system? Or do we want to know the average current and voltage in a system? You can do the calculations for any of these scenarios. For example, if we want to know the maximum power generated by a generator, we could write:

$$P_{max} = I_{max} \Delta V_{max}$$

Be careful not to mix maximum values and average values. You would never multiple the average current by the maximum voltage, or vice versa. Finding the average value is more complicated because the functions are oscillating.

1. What is the average value of a sine function that oscillates around zero? Can you tell the difference between the two function shown at right using the average?



2. Given this, what is the average value for current and voltage in an AC system?

This is a problem. We need a way to describe the average *magnitude* current or voltage in a circuit. To address this concern, we have two options: (1) we could take the absolute value of the function or (2) we could square the function. Let's look at an example.

3. What is the average of each of the following two lists of numbers?

-10	-8	-6	-3	-1	1	3	6	8	10
-100	-80	-60	-30	-10	10	30	60	80	100

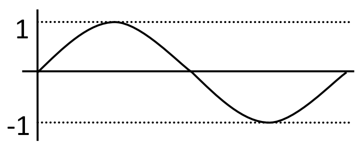
4. This average does not show that the second list is larger in a magnitude or absolute value sense.

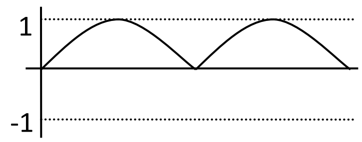
   a) Find the average of these two lists using the absolute value of the numbers.

   b) We can also get rid of the minus signs by squaring all the numbers before we average. Find the average by squaring all the numbers, then average these squared numbers, then take the square root of this average.

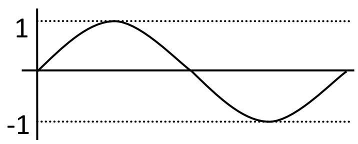
Let's look at a sine graph with amplitude 1 to compare the two methods.

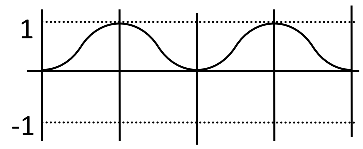
Taking the absolute value of the sine function looks like the graph on the right.





Squaring the sine function looks like this.





5. The first graph is difficult to average but the second is less troublesome. Use the fact that the dips are exactly the same shape and size as the humps to determine the average of the sine function squared.

6. Because we squared the sine function, we need to now take the square root. What is the result?

The standard for reporting average current and voltage in AC circuits is called Root-Mean-Squared (rms). This is what you just did above. The function is squared, then averaged (take the mean), and finally we take the square root. For the sine function above with amplitude 1, you found that the rms value was $\frac{1}{\sqrt{2}}$. This means for current and voltage, which oscillate sinusoidally, we can write the following relationships:

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} \qquad \Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}}$$

Now let's use these relationships to find the average power delivered:

$$P_{avg} = I_{rms}\Delta V_{rms}$$

$$P_{avg} = \frac{I_{max}}{\sqrt{2}}\frac{\Delta V_{max}}{\sqrt{2}} = \frac{P_{max}}{2}$$

So we see that the average power delivered in an AC circuit is half the maximum power (sometimes called the peak power).

7. Usually, the current and voltage values reported for AC systems are the rms values. If the US runs on 120 V rms, what is the peak voltage for your household circuits?

8. I have a toaster running in my kitchen that has a resistance of 500 Ω. How much rms current does it draw? What is the peak current?

9. What is the peak power output for my toaster? What is the average power output for my toaster?

Based in part on an activity developed by Larry Watson for the Physics Learning Center at the University of Wisconsin-Madison.

Lab: Transformers

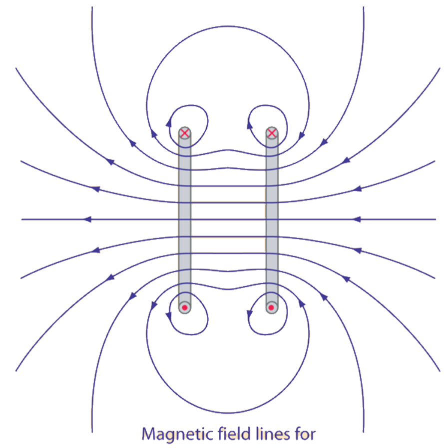
Transformers are key to the modern AC electrical grid system. They take advantage of Faraday's Law to change the voltage in a system. In this laboratory investigation, we will explore how transformers work to generate electricity through induction.

Part 1: Helmholtz Coils

For this experiment, you will be using set of Helmholtz coils to generate the magnetic field. Helmholtz coils are an arrangement of two electromagnetic coils in which the distance between the coils is the radius of the coils. This set-up produces a nearly constant magnetic field in the center of the system, as shown in the figure. The relationship between current and magnetic field in the center of a Helmholtz coil is given by:

$$B = \frac{8}{5\sqrt{5}} \frac{\mu_0 N I}{R}$$

Where $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Tm/A}$, N is the number of turns of wire in each coil, I is the current through the wires, and R is the radius of the loop.



Magnetic field lines for Helmholtz coils.

1. Look at the expression above. What happens to the strength of the magnetic field as the current is increased?

2. If the current oscillates sinusoidally as a function of time, what does the magnetic field do as a function of time?

3. In a generator, the magnet was moving through the coils, which caused a changing magnetic field. In this experiment, the small coil is stationary and sitting inside the large coil. Under what circumstances will a voltage be induced in the small coil? Explain.

4. What factors will determine how much voltage is induced in the small coil?

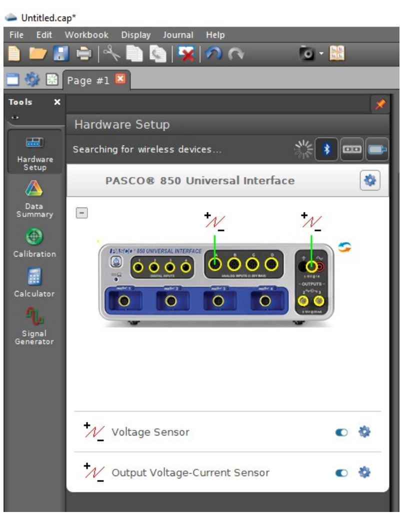
5. How is the AC voltage different from the DC voltage? What does this mean for the magnetic field that is generated inside the Helmholtz coils?

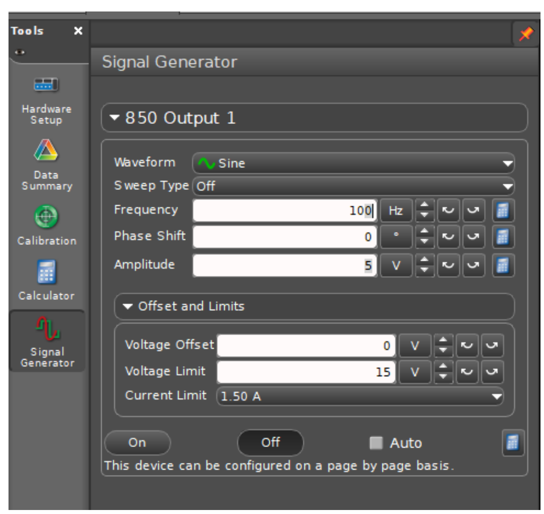
Part 2: AC Induction

For the experiment, we will put a smaller coil inside of the larger Helmholtz coils. The smaller coil will be connected to a voltage sensor that can detect any induced voltage in the coils. The larger Helmholtz coil will have AC current running through it, which will be controlled using the computer interface.

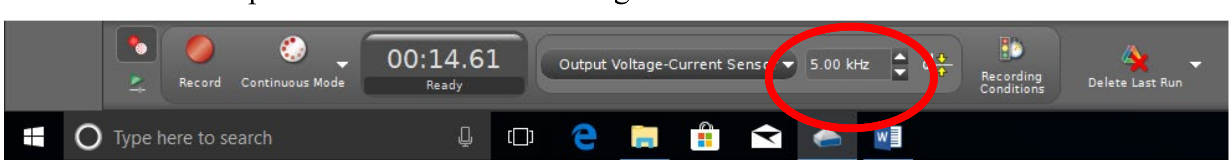
Set up the system with the following specifications:

- Connect the Helmholtz coils in parallel to the power supply on the Pasco Universal Interface.
- Connect the smaller coil to a voltage probe. Plug the voltage probe into the Pasco Universal Interface.
- Use the Hardware Setup button on the left of the screen to activate the power supply and add the voltage probe, as shown in the figure below (left).
- Add two graph displays to show the voltage vs. time. One is for the Helmholtz coils (select Output Voltage), and the other is for the smaller coil (select Voltage Sensor).





- Use the Signal Generator to select the Sine waveform. Set the Amplitude (voltage) to 5.0 V and a frequency of 100 Hertz. (Shown above, right.)
- At the bottom of the screen, change the sample rate to 5.0 kHz. Select "Common Rate" from the dropdown menu. This will change is for all devices.



6. Place the smaller coil in the middle of the Helmholtz coils and start recording data. What do you observe about the voltage generated in the small coil?

We're going to take some measurements of how the maximum voltage generated depends on various factors. The maximum voltage is the amplitude of the sine wave you observe in the voltage vs. time graph. We'll do two sets of measurements to determine:

- How does voltage in the small coil depend on the voltage in the Helmholtz coil?
- How does voltage in the small coil depend on the number of turns in the small coil?

As you collect data, fill in the data tables below. After collecting the data, make a graph in Excel and determine the relationship between these two factors.

Experiment #1: How does Helmholtz voltage affect small coil voltage?

What will you hold constant in this experiment? ______________________________________

Helmholtz Voltage (V)	**Small Coil Max Voltage (V)**
4.0 V	
6.0 V	
8.0 V	
10.0 V	
12.0 V	
14.0 V	

Claim that you can make based on this data:

Experiment #2: How does the number of turns of wire affect small coil voltage?

What will you hold constant in this experiment? ______________________________________

Number of Turns of Wire	**Small Coil Max Voltage (V)**
200	
400	
800	

Claim that you can make based on this data:

7. Based on these observations, what is the relationship between voltage generated in the small coil and the factors we investigated? Write a mathematical expression to describe the relationship.

8. This system is effectively a transformer – it is converting one voltage to another through electromagnetic induction. How does this relationship compare to what we saw with the generators? What are the similarities and differences between the two systems?

Part 3: Transformers

Transformers have two set of coils sharing an iron core. Typically, one set of coils is inside the other. The iron core serves to magnify the magnetic field and increase the flux in the system. The transformer that we will use for this experiment is shown below. The two coils are sharing an iron core, but in a slightly different arrangement.

Set up your coils as shown in the photograph. Connect one side of the coils to the power supply in the Pasco Universal Interface. The coil connected to the power supply is called the **primary coil.** Connect the other side to the Voltage probe, which will measure the voltage. The coil that has an induced voltage is called the **secondary coil**.

Our goal in this activity is to test the model you developed above on a slightly different system. In particular, we want to improve the model by including the number of coils (N_1) in the equation.

1. Design an experiment that you can do with the equipment provided to test the mathematical model you found above. Record your procedure and data below.

2. Did your model hold for this system? If not, what changes would you make to the model?

Now, we'll develop a theoretical model and compare to the empirical model you found above. We can apply Faraday's Law to each of the coils in this system:

$$\Delta V_1 = -N_1 \frac{\Delta \Phi_B}{\Delta t}$$

$$\Delta V_2 = -N_2 \frac{\Delta \Phi_B}{\Delta t}$$

3. For an ideal transformer, the coils are sharing an iron core, which means that the rate of change in flux would theoretically be the same in each side. Given this assumption, what mathematical relationship can you derive that shows the relationship between the voltage and the number of turns in each coil?

4. How does the theoretical expression compare to what you found experimentally? Explain any discrepancies.

5. For an ideal transformer, the power in from the primary coil is the same as the power out by the secondary coil. This is due to energy conservation in the system. How could you test to see if this is true for your transformer? (Hint: recall that $P = I\Delta V$.)

Reading: The Electric Grid

The US electricity system, which precisely balances supply and demand while delivering electromagnetic energy that propagates at nearly light speed, has often been described as the most complex machine every built. Indeed, in 2003 the US National Academy of Engineering named the electricity system as the greatest engineering achievement of the 20th century. The academy notes that the system is ingeniously engineered; it is a catalyst for new technologies and new industries; and it has an enormous impact on improving the quality of life. It is arguably the most influential machine of the 20th century.[4]

(Gellings &Yeager, 2004)

The **electrical grid** is the vast network of electrical transmission lines that bring electricity from the power plan to homes and businesses. In the US, there are three interconnected grid systems, one for the Eastern US, one for the Western US, and one in Texas. Many power plants run on fossil fuels, such as coal or natural gas, but alternative sources of electricity, such as wind, hydropower, and nuclear, are also included in the electrical grid. All of these power plants (with the exception of solar) generate alternating current (AC) electricity. (Photovoltaic cells are connected to a device called an inverter, which converts DC power to AC.)

A power plant can generate voltage between 2,000 and 22,000 AC Volts. This voltage is then stepped up by a transformer to be transmitted via power lines to local communities. The electricity in transmission lines runs at a much higher voltage, typically 345,000 Volts. Another transformer at your neighborhood substation will step the electricity down to 2,400 Volts, which is further stepped down to 120 Volts (rms) before entering your home. If you live in the country, you might have a green transformer in your front yard. If you live in the city, there is likely a transformer on the utility pole.

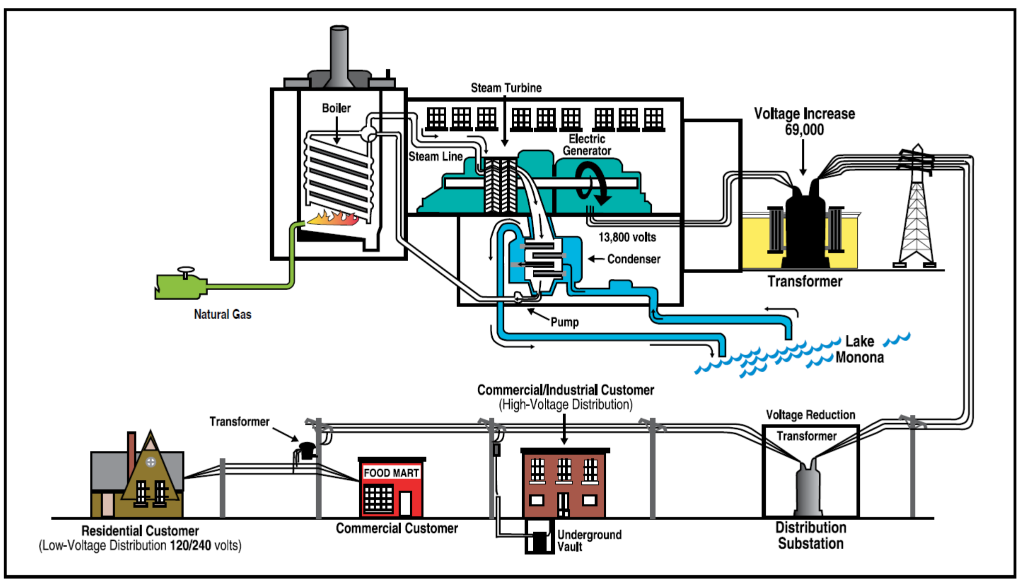


Figure 1: A schematic of the electric grid (Image credit: Madison Gas & Electric)

[4] Gellings, C. W. & Yeage, K. E. (2004). Transforming the Electric Infrastructure. *Physics Today*. Dec 2004: 45-52.

Our electric grid system is amazing, but the grid is also aging, much of it having been built in the 1950's. We take electricity for granted, until it's not there. In August 2003, the United States experienced the worst blackout in the country's history. Much of the Northeast US and parts of Canada were without power for up to four days. A 2013 *Time Magazine* article reflects on the disaster:

> It wasn't the tree's fault — or least, not just the tree's fault. Nearly 10 years ago, on Aug. 14, 2003, the electricity grid in the U.S. Northeast was stressed close to the limit. This wasn't unusual; summer is a period of high demand in the Northeast, as air conditioners run overtime to compensate for the heat, and a number of older power plants were already offline for maintenance. As power lines became overloaded, they began sagging because of the high temperatures, until one line south of Cleveland touched an overgrown tree limb and short-circuited. What followed was a cascade of disaster due to a mix of human error and equipment failure, until by 4:10 p.m. E.T. that day more than 50 million people had lost power in parts of Ontario and eight U.S. states. New York City looked like this [Figure 2], and power wasn't fully restored for two days. At the time it was the second most widespread power blackout in history, after a 1999 disaster in Brazil. … The blackout was a big deal, leading to at least 11 deaths and costing the economy some $10 billion. More important, the disaster underscored just how rickety our interconnected and jury-rigged electrical grid was — and how vulnerable it could be to disruption, both accidental and malevolent.[5]

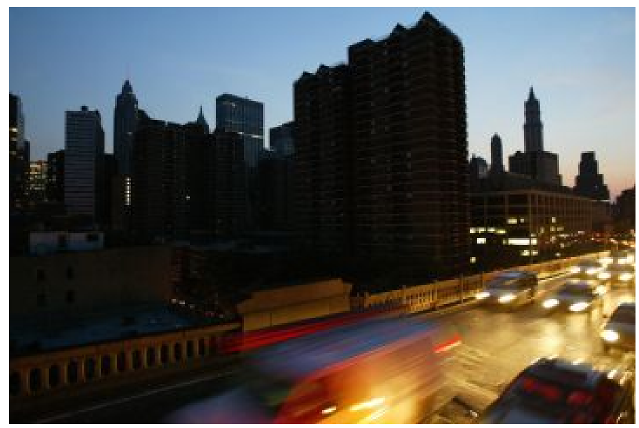


Figure 2: New York City during the 2003 Blackout
(Image credit: [Time Magazine])

A functioning electric grid is critical to modern society. One of the constant challenges is that we do not have an efficient way to store electricity. This means that electricity must be used immediately after being generated. Engineers at power plants build models based on past usage to predict how much electricity will be needed on a particular day and at a particular time.

Figure 3 shows what engineers call a **power demand curve**. This curve shows typical electricity usage peaks in the late afternoon and early evening. Electrical engineers have curves like this for every day of the year, which can help them to determine how much energy will be needed on a particular day. In the summer, electricity demands are typically large due to air conditioning systems. To meet this demand, many power companies will run "peaker plants" that only run for a few months each year. This is expensive to maintain, but is the only way we currently have to consistently meet demand.

To see how much electricity is being used in real time, check out the website for Midcontinent Independent System Operator (MISO), an independent organization that provides power for much of the Midwest. The link to their real-time display is given below:

https://www.misoenergy.org/markets-and-operations/real-time--market-data/real-time-displays/

[5] Walsh, B. (2013). 10 Years After the Great Blackout, the Grid Is Stronger — but Vulnerable to Extreme Weather. *TIME Magazine*, Aug 13. 2013. Available online: http://science.time.com/2013/08/13/ten-years-after-the-great-blackout-the-grid-is-stronger-but-vulnerable-to-extreme-weather/

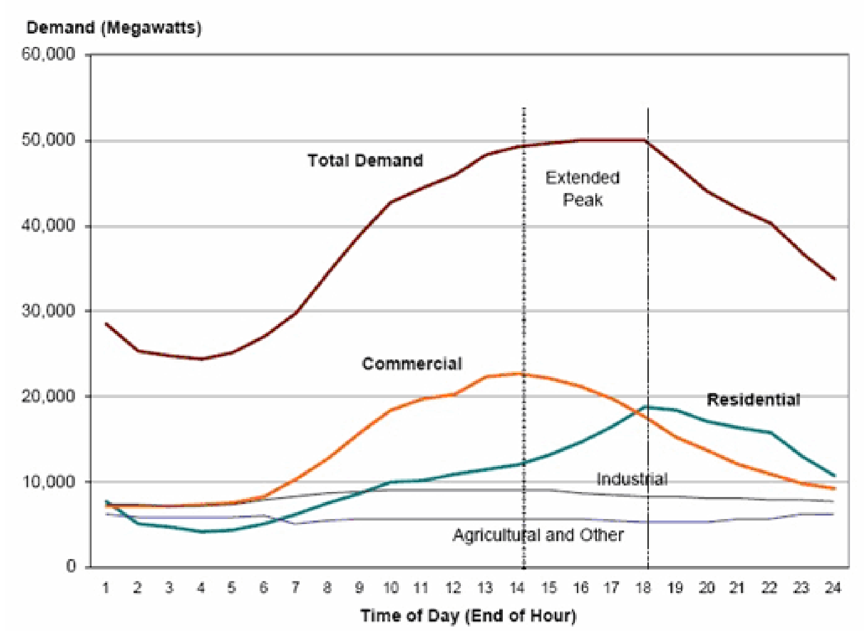


Figure 3: Power demand curve. (Image credit: Power UK)

Engineers are working on ways to improve the nation's aging electrical infrastructure. New designs for the grid will integrate better with alternative energy, such as wind, solar, and nuclear energy, as well as employing smart technologies to regulate power usage. Other solutions involve improving federal regulations for maintenance, which were implemented following the 2003 blackout. The results are described below in the *Time Magazine* article:

> The Federal Energy Regulatory Commission (FERC) now has the ability to impose fines of up to \$1 million per violation per day for failure to comply with those standards. Beyond that, though, utilities invested real money to make the grid more resilient. In an analysis conducted for the Associated Press, the software and data service firm Ventyx found that utilities spent an average of \$21,514 per year on devices and station equipment per mile of transmission line from 2003 to 2012 — nearly three times what they spent from 1994 to 2003. Maintenance spending for overhead lines increased by an average of 8.2% per year from 2003 to 2012, compared with just 3% a year on average from 1994 to 2003. Thanks in part to \$4.5 billion in federal stimulus money allocated toward the construction of a smart grid, utilities have been able to add hundreds of advanced grid sensors and millions of smart electrical meters, which help power companies keep near real-time tabs on the state of the grid. And it doesn't hurt that power demand has remained flat or fallen over the past decade, as devices and appliances became more efficient and economic growth slowed down.

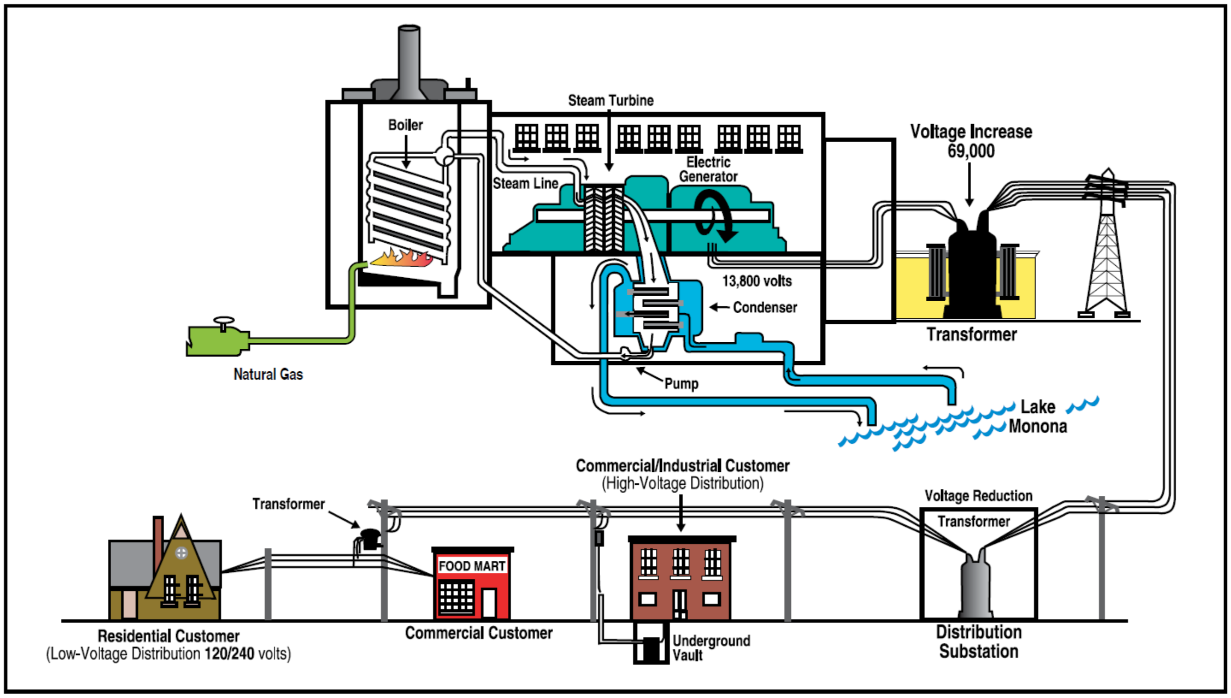
These changes implemented since the 2003 blackout have made the grid more robust. The average amount of time a consumer spends without electricity has decreased considerably since then, reflecting a more reliable electrical grid.

However, having such a large grid makes us vulnerable to natural disasters or terrorist attacks. Some engineers advocate for a different approach consisting of smaller "microgrids" that would be a system of interconnected cells of electricity generation on a local level. This plan would include more renewable energy, such as solar and wind, but would also require a system for storing electricity. Although cost prohibitive now, battery technology is advancing, which would make microgrids a feasible solution in the future.

Tutorial: Blount St. Power Plant

The power grid is the network of electrical lines running across our country. It is what connects homes, schools, and business to the power generation stations, such as those run by Madison Gas & Electric (MGE). In this tutorial, we will explore how this system works, the limitations of the current system, and ideas for improving it in the future.

Part 1: Efficiency of the Power Plant



1. Above is a diagram of MGE's Blount St. power plant. Use the diagram to identify the energy transformations occurring in the system, starting with the natural gas and ending at your home.

2. Natural gas is 95% methane. In a combustion reaction, methane releases 5.55×10^7 J/kg of energy. If 8,500 kg of natural gas is burned each hour, how much energy is generated in this time period?

The two boilers in the Blount St. plant produce 400,000 lbs (9.7×10^4 kg) of steam per hour at 950ºF (783 K). Blount St is a peaking plant, so it is used most often in the summer. Given this, let's assume the water coming in is at 300 K.

To calculate the total amount of energy needed to heat this water into steam we will have to revisit some chemistry concepts. The specific heat of a substance is the amount of heat needed to raise the temperature of 1 kg of a substance by one Kelvin. The latent heat of vaporization is the amount of heat needed to vaporize a substance (i.e. turn water into steam). The total amount of heat needed to raise the lake water to 950ºF would be given by the following expression:

$$Q = (mc\Delta T)_{water} + mL_v + (mc\Delta T)_{steam}$$

This means that we need to add energy into the water to heat it to boiling (T = 373 K), then turn the water to steam, then heat the steam from 373 K up to the maximum temperature of 783 K. (During the time when water is converted to steam there is no change in temperature.)

3. Use the data table at the right to calculate the total amount of energy required to heat the steam.

Helpful constants	
Specific heat of water	4186 J/kg-K
Specific heat of steam	2490 J/kg-K
Latent heat of vaporization (water → steam)	2.26×10^6 J/kg

4. What accounts for the difference between the answers you got for #2 and #3?

5. The steam's kinetic energy is then used to spin the blades of the turbine and power the generator. Due to thermodynamic inefficiencies, about half of the energy needed to heat the water to steam is transferred to the turbine. How much kinetic energy does the turbine have? What happens to the rest of the energy? The generator is then up to 99% efficient at converting kinetic energy to electrical energy.

6. The above calculations were done to find the total kinetic energy of the system over the course of an hour. Convert these energy calculations to power and fill in the table below.

	Type of energy	Energy (J) in one hour	Power (W)	Efficiency from one step to the next
Natural Gas				
Boiling water to steam				
Generator spinning				
Output of generator				

7. Finish the table above and calculate the efficiency from one step to the next $\left(e = \frac{P_{out}}{P_{in}}\right)$. What is the total efficiency of the system from natural gas to electricity produced by the generator?

8. How could the efficiency of this system be improved? What could you do with all of the thermal energy that is wasted?

9. Blount Station has two 50 MW generators. Given this, what is the total power output of the plant? If one megawatt (1 MW = 10^6 W) provides electricity to about 200 homes, how many homes can be serviced by this power plant?

10. The voltage produced by the power plant is 13,800 V. Given this, what is the current when the electricity leaves the generator?

Part 2: Transmission Lines

Transmission lines are what carry the electricity from the power plant to your house. A transmission line is essentially a big wire, which means it has some resistance. This means that the transmission line acts like a big resistor and heats up like a resistor. In other words, the electrical energy is converted into heat; this is known as **Ohmic heating**. We can figure out how much power is lost due to this effect by substituting Ohm's Law ($\Delta V = IR$) into the power equation:

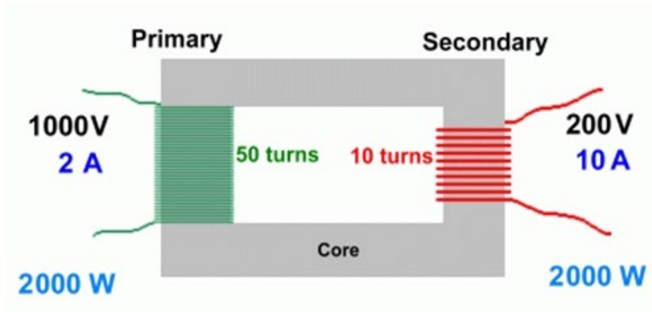
$$P_{lost} = I\Delta V = I^2 R$$

1. If we assume a transmission line has a resistance of 0.05 Ω per km. How much power is lost over the 30 km transmission line from the power plant to your house? What percentage of the total power is lost over this distance? (Use the current you found in #10 above.)

This is a problem! We can't be losing such a high percentage of the energy! Transformers to the rescue. (More than meets the eye…)

A transformer is a device that can change the voltage in a given system. A "step up" transformer converts a low voltage to a high voltage. A "step down" transformer converts a high voltage to a low one.

A transformer is essentially two coils of wire wound around the same iron core. Because of energy conservation, the power (the rate that energy is generated) in the primary coil must be the same as the power through the secondary coil.



$$P_1 = P_2$$

$$I_1 \Delta V_1 = I_2 \Delta V_2$$

2. As the electricity leaves the power plant, it goes through a step-up transformer. Looking at the equation above, what happens to the current as we increase the voltage of the system? How will this affect the power lost over the transmission lines?

3. The 13,800 V electricity that leaves the power plant is increased to 69,000 V. What is the new current at this voltage?

4. Let's look at the same transmission line (resistance of 0.01 Ω per km). How much power is lost over the 30 km transmission line from the power plant to your house? What percentage of the total power is lost over this distance?

The voltage is stepped back down again when it gets to your house. If you live in the country, the transformer is in the green box that might sit in your front yard. If you live in the city, there is a neighborhood substation. In the US, residential voltage is 120 V. The power also gets split up to go to multiple houses.

5. A typical house in Madison draws about 5000 Watts of power. Given this, what is the current coming out of the transformer and going into your house? Why does it need to be so high? (Hint: are the circuits in your house wired in series or parallel?)

Part 3: Disaster

On July 19, 2019, a transformer at MGE's Blount St. power plant caught on fire. It was a record-breaking hot day, $98^{o}F$ with a heat index of $108^{o}F$. The fire at MGE was actually the second explosion of the day, the first was at a substation on Dayton St., near the University of Wisconsin campus.

1. What could cause a transformer to catch on fire or explode?

2. The two disasters were related, even though the locations were mile apart. How could one transformer failure lead to another at a different location?

3. How might the hot weather have led to the disaster?

4. The power company decided to cut power to a large portion of the city while repairs were made. Do you think this was the right decision? Explain your position.

Image credits: Steve Apps, [Wisconsin State Journal](#) (top); Madison Fire Department, [WKOW](#) (right)

Case Study: Niagara Falls & AC Power

This case study returns to the story of harnessing the power of Niagara Falls. We'll pick up the story here where the engineers were debating how to transmit the electricity from the falls into the city of Buffalo. The readings below are excerpted from an article by Jack Foran called "The Day They Turned The Falls On: The Invention Of The Universal Electrical Power System"[6]

Review the first reading (page 4) in this book to refresh your memory.

1. What was the problem with transmitting power using DC? What is the role of Ohm's Law ($\Delta V = IR$) in this?

2. Why can't a transformer be used to help transmit DC power?

3. What were the initial drawback of AC? Why were people slow to realize the advantages of AC power?

The State of the Art

In an endeavor to gain more expertise on the power production and transmission matters, in 1890, [Edward Dean] Adams went to Europe to observe power installations and consult with the engineers who had built and operated them. Prior to his departure, he sold his shares and resigned his directorship in the Edison company, so as to eliminate any vested interest he might have toward one solution or another. He traveled in England, France, Germany, and particularly Switzerland, where hydropower development was reputed to be most advanced, and by early summer had reached two conclusions.

First, if transmission was the key to making the project viable, then all the power could be produced at one location. The multiple power-production sites would be unnecessary. This would eliminate the dozen inlet canals over the mile-and-a-half stretch of upper river shoreline, the 238 wheel pits and individual turbines, and a mile and a half of the discharge tunnel. For a central station just upriver of the reservation, only a mile of tunnel would be necessary. The tunnel would still be a gargantuan project, but shortening it by about two-thirds would reduce construction costs enormously.

Second was an idea for an international competition among power-production companies and experts to determine the best solution to the means of power for the Niagara situation. The other principals in the project thought this was a good idea and an International Niagara

[6] Published by the National Center for Case Study Teaching in Science (http://ublib.buffalo.edu/libraries/projects/cases/niagara.htm)

Commission was set up to conduct and judge the competition. The five-man commission was to be headed by the pre-eminent British mathematician and physicist of the day, William Thomson, whose numerous theoretical and practical scientific contributions--from establishing the law of conservation of energy to the laying of the first transatlantic telegraph cable--would result in his knighthood as Lord Kelvin.

The commission issued invitations for proposals in three categories: power development; transmission and distribution; and a combination of the first two categories. Seventeen projects were submitted, of which three were dismissed as not complying with the terms of the invitation or insufficiently complete to warrant judging. For power generation, all but one proposal used some type of rotary turbine or water wheel. The exception was for a series of underground pistons, driven by the weight of the water column above them, that would function as air compressors.

For transmission and distribution, seven of the 14 proposals were for electricity. Five were DC and two AC. The DC proposals were for transmission to Buffalo at up to 16,000 volts, typically with receiving motors in Buffalo and secondary dynamos for regeneration at lower voltages for distribution. One of the proposals was for 5000-volt transmission to charge storage batteries in Buffalo for redistribution.

One of the AC proposals was for polyphase, that is, two separate currents or waves of electricity (since AC takes the form of a sine wave) out of phase. The polyphase proposal called for step-up and step-down transformers. The electricity would be generated at 2000 volts, stepped up to 10,000 volts for transmission, and then in Buffalo stepped down to 2000 volts or lower, depending on intended use.

Four of the proposals were for compressed-air transmission through underground mains, usually about two feet in diameter. The compressed-air proposals touted the purported advantages of this medium, for example, its applicability to basic industrial uses such as hauling and lifting (one of Buffalo's chief industries was transshipment of grain, and in this regard Buffalo had invented the grain elevator, a mechanism to transfer grain from lake ships to storage prior to retransfer to mills or further modes of transportation). The proposals also noted that steam engines, which were the ordinary form of industrial mechanical power at the time, could readily be utilized--practically without conversion--as compressed air engines. One of the proposals included a trolley line to be constructed over the air main and driven by compressed air.

One hydraulic transmission scheme was proposed. It called for ten 2-foot diameter underground mains and pressure pumps, all of which the commissioners considered rather cumbersome. One proposal was for mechanical transmission via steel cables in a chain of posts and pulleys. Generation was 8000 horsepower, which it was claimed would lose only seven horsepower per 330-foot span. And one proposal mentioned various forms of distribution, including electric, compressed-air, and hydraulic, but gave no particulars.

Eight prizes were awarded, but no top prize for a plan combining power production and distribution. The commission report noted that "There was no project which, in the opinion of the Commission, could be recommended for adoption without considerable modification."

Neither Edison nor Westinghouse submitted a project to the commission. Edison originally had been suggested as a commission member, but then was not asked, possibly because he might be expected to submit a proposal or to be involved in the development of the project.

Lewis B. Stillwell, a Westinghouse engineer who would later become the electrical director of the Niagara Falls Power Company, wanted to submit a proposal, but Westinghouse rejected the idea on the theory that "these people are trying to get $100,000 worth of information for a prize of $3000. When they are ready to do business, we will submit a plan and bid for the work." Which is just what he did.

4. How did solving the transmission problem help to reduce construction costs?

5. One proposal recommended transmitting DC power at 16,000 V from Niagara to Buffalo. If the cables have a resistance of 0.05 Ohm/mile, how much energy would be lost each second during the transmission? The distance between Niagara and Buffalo is about 20 miles.

6. Another recommended producing AC power at 2000 V, then transmitting it at 10,000V to Buffalo. How was this proposal different from the one mentioned above?

7. A hydraulic transmission proposal claimed that it would only lose 7 horsepower per 330 feet. How does this compare to the energy loss calculated in above for the 20 mile trip from Niagara to Buffalo? (1 hp = 745 W; 1 mi = 5280 ft)

Nikola Tesla--The Father of AC

But instead of resolving the matter of the best means to convey Niagara power to Buffalo, the International Niagara Commission had left it as confused as ever. Most of the plans submitted were for DC, but DC had known drawbacks for transmission. A close second, in terms of numbers, to DC was compressed air. Several plans were for AC, and the most elaborated and intriguing of these was for polyphase AC. The polyphase AC plan had garnered no prizes, however.

The problem of the best means of transmission, though, would be worked out not by the commission but in the natural course of things, which included great strides in the development of AC. In addition, the natural course of things included some special intervention from on high (that is, from Edison himself).

But above all, it involved Tesla, probably the only inventor ever who could be put in a class with Edison's in terms of the number and significance of his innovations. The Croatian-born scientific mystic--he spoke of his insight into the mechanical principles of the motor as a kind of religious vision--had once worked for Edison. He had started out with the Edison Company in Paris, where his remarkable abilities were noticed by Edison's business cohort and close friend Charles Batchelor, who encouraged Tesla to transfer to the Edison office in New York City, which he did in 1884. There Edison, too, became impressed with him after he successfully performed a number of challenging assignments. But when Tesla asked Edison to let him undertake research on AC--in particular on his concept for an AC motor--Edison rejected the idea. Not only wasn't Edison interested in motors, he refused to have anything to do with the rival current.

So for the time being Tesla threw himself into work on DC. He told Edison he thought he could substantially improve the DC dynamo. Edison told him if he could, it would earn him a $50,000 bonus. This would have enabled Tesla to set up a laboratory of his own where he could have pursued his AC interests. By dint of extremely long hours and diligent effort, he came up with a set of some 24 designs for new equipment, which would eventually be used to replace Edison's present equipment.

But he never found the promised $50,000 in his pay envelope. When he asked Edison about this matter, Edison told him he had been joking. "You don't understand American humor," he said. Deeply disappointed, Tesla quit his position with the Edison company, and with financial backers, started his own company, which enabled him to work on his AC ideas, among other obligations.

The motor Tesla patented in 1888 is known as the induction motor. It not only provided a serviceable motor for AC, but the induction motor had a distinct advantage over the DC motor. (About two-thirds of the motors in use today are induction motors.) The idea of the induction motor is simplicity itself, based on the Faraday principle. And its simplicity is its advantage over the DC motor.

An electrical motor--whether DC or AC--is a generator in reverse. The generator operates by causing a conductor (armature) to move (rotate) in a magnetic field, producing a current in the armature. The motor operates by causing a current to flow in an armature in a magnetic field, producing rotation of the armature. A generator uses motion to produce electricity. A motor uses electricity to produce motion.

The DC motor uses commutators and brushes (a contact switching mechanism that opens and closes circuits) to change the direction of the current in the rotating armature, and thus sustain the direction of rotation and direction of current.

In the AC induction motor, the current supply to the armature is by induction from the magnetic field produced by the field current. The induction motor thus does away with the troublesome commutators and brushes (or any other contact switching mechanism). However, in the induction motor the armature wouldn't turn except as a result of rotation of the magnetic field, which is achieved through the use of polyphase current. The different current phases function in tandem (analogous to pedals on a bicycle) to create differently oriented magnetic fields to propel the armature.

Westinghouse bought up the patents on the Tesla motors almost immediately and set to work trying to adapt them to the single-phase system then in use. This didn't work. So he started developing a two-phase system. But in December 1890, because of the company's financial straits--the company had incurred large liabilities through the purchase of a number of smaller companies, and had to temporarily cut back on research and development projects--Westinghouse stopped the work on polyphase.

8. What started the rift between Edison and Tesla?

9. What was the advantage of the AC motor over the DC motor?

10. How did Tesla's invention spur the development of polyphase (three-phase) current?

11. How is a motor like a generator? How are they different?

12. What was the role of Westinghouse? Whose idea did he support?

The War of the Currents

The War of the Currents, which had begun as a spirited but more or less conventional exchange of business propaganda claims and counterclaims, heated up in 1888 with the publication of Edison's "A Warning" on the mortal danger of AC. What drove Edison to this extreme measure must have been in part a genuine belief that AC was more dangerous than DC, but also a vague sense that something was catching up on him. Edison still had more installations than Westinghouse, but the Westinghouse and other AC companies were growing faster than Edison's companies. No doubt, a sharp rise in the price of copper in 1887 contributed to Edison's mood. A French syndicate had cornered the copper market and driven the price up. DC systems, because of the low voltages, used thick copper conductors to lessen resistance, whereas AC could be sent on thin wires at high voltages. So the copper problem hurt Edison more than it hurt the AC systems.

Coincidentally, about this time the New York State Legislature was looking for a more humane, or at least more efficient, way to execute criminals. They considered numerous methods, including electricity. When they asked Edison's opinion, he said he thought electricity would do the job "in the shortest space of time, and inflict the least amount of suffering upon its victim." And as apparatus, he recommended "alternating machines, manufactured principally in this country by Geo. Westinghouse." He pointed out that "the passage of current from these machines through the human body, even by the slightest contacts, produces instantaneous death."

…

[O]n August 6, 1890, at Auburn prison, in a grisly ceremony, Kemmler was executed. The first application of current, lasting 17 seconds, didn't complete the job. The smell of burned flesh caused spectators to become nauseous, but when the electricity was turned off, they noticed a slight heaving of the victim's chest. "Good God, he is alive," one man said. A press representative fainted. A second current was applied that according to the New York Times account lasted anywhere from one minute to four-and-half minutes, since witnesses with watches had been too horrified to check them. This current had the intended effect. But in the aftermath it was disputed whether Kemmler died of electric shock or was simply "roasted to death." The Times described the event as "an awful spectacle, far worse than hanging."

The Triumph of AC

But by and large, Westinghouse--or at least the Westinghouse Company--seemed to focus its energy where it would do the most good, namely, in the development of an efficient and effective AC system.

In 1890, Westinghouse installed a 12-mile, 4000-volt transmission line from Willamette Falls to Portland, Oregon. And in Telluride, Colorado, in 1891, the company installed the first transmission line for electricity for power rather than just for lighting. The transmission distance was just three miles, but at the end of the line, the electricity was used to operate a 100-horsepower synchronous motor, in conjunction with a Tesla induction motor to start the synchronous motor.

Meanwhile, even more impressive AC accomplishments were being achieved in Europe. Principal among these was the long-distance transmission in August 1891 from Lauffen to Frankfurt am Main, a distance of 100 miles, of three-phase power at 25,000 volts. The electricity was used at an International Electrical Exhibition to provide lighting and run a wide array of machinery and even a small artificial waterfall, symbolic of the source of the power.

Adams says that in early fall of that year--just after the Lauffen-to-Frankfurt achievement--it became clear that AC electricity would be appropriate for the transmission aspect of the Niagara project. So that electricity would do for the whole project, with AC for transmission to Buffalo, DC for Niagara Falls. Prior to this time the consensus had developed to go with DC locally and a compressed-air system to Buffalo. Two powerhouses were planned (and eventually built), and the plan had been that Power House No. 1 would be for electricity, and Power House No. 2 for compressed air.

In December 1891, the Cataract Company issued an invitation to six companies for design and construction of the electrical installation for the Niagara project. It didn't mention AC or DC. … About this time, the Westinghouse Company decided to push development of the two-phase system. Work was resumed on the induction motor and pursued on the rotary converter. The rotary converter, which was used to convert AC to DC, was essentially a combination of an AC motor operating a DC generator.

Also about this time, bids were requested for lighting the Columbian Exposition to be held the next year in Chicago. Westinghouse purposely underbid the project, wanting it for its enormous publicity value. His bid was $399,000 for a polyphase system. Edison's bid was more than $1,000,000. Westinghouse won the job. …

The Columbian Exposition opened in May. Westinghouse had taken full advantage of his opportunity. Not only did he light the fair in spectacular fashion, but the electrical system used and displayed was a prototype of the universal system. Using a 1000-horsepower generator, the system included step-up and step-down transformers to demonstrate the principles of transmission, induction and synchronous motors, and rotary converters producing DC for arc lights and streetcars. The Chicago fair installation was said to remove the last objection to AC. About this same time, also, the Westinghouse Company completed a 10,000-volt, 35-mile transmission line using step-up and step-down transformers to supply current to Pomona, California.

Figure 1: The Colombian Exposition (Image credit: PBS)

13. How did the price of copper affect the development of AC and DC transmission?

14. What drastic measures did Edison take to convince the public of the dangers of AC power?

15. How did Westinghouse gain public support for AC?

16. What does this tale about Edison, Westinghouse, and Tesla tell us about scientific practice and the development of new technologies? Is it always objective and based on data and evidence?

Request for New Proposals

In August 1893, following Forbes' preliminary redesign of the generator, the Cataract Company issued an invitation for new electrical proposals from General Electric and Westinghouse. The new design Forbes had come up with called for the field magnets to revolve outside a stationary armature. The purpose of the new design was to provide an improved flywheel effect. In addition, Forbes' generator had a frequency (which is the number of current alterations, or cycles, per second) of 16 2/3, and generation at 20,000 volts.

The invitation for new proposals stated that the preliminary design of the generators was now sufficiently complete to allow proper bids. The invitation also noted that "any alterations that you may propose in the design will be carefully considered, and if acceptable, will be appreciated in awarding the contract."

The new request for proposals elicited an irate letter from Westinghouse, who may have suspected--just because Forbes had ventured a redesign--that the Niagara company was contemplating designing or even building the apparatus in-house, and merely wanted the benefit of his company's ideas, free of charge. In addition, Westinghouse engineers objected to Forbes' idea to generate at such high voltage (20,000) and such a low frequency (16 2/3). They felt that in both these matters Forbes had failed to recognize key AC advantages, namely, the use of transformation to boost voltage for transmission, and the use of the induction motor. The main problem with generating at 20,000 volts would be with insulation of the generating apparatus. The main problem with the 16 2/3 frequency was that it would be disadvantageous for running high-speed induction motors.

The Westinghouse Company won the contract. Despite the objections. Or perhaps because of them. The Niagara company apparently meant what it said about appreciating proposed alterations. The umbrella-type generator was used, but the generating voltage was set at a more reasonable 2200, and a compromise was reached on frequency at 25 cycles. The initial contract was for the three 5000-horsepower generators. Power was first produced in Niagara Falls on August 26, 1895.

The GE and Westinghouse proposals for the transformers and transmission apparatus were also virtually identical, and GE got the contract (the J.P. Morgan connection may have helped). The first transmission to Buffalo occurred on November 15, 1896, with attendant hoopla, including the firing of a 21-gun salute by the Ninth Ward Polish-American Gun Squad over the Niagara River outside the Buffalo power station. The transmission was at 11,000 volts, three-phase, and the power was purchased by the Buffalo Railway Company, which operated the city trolley system.

The invitation for bids on design and construction of the generators in December 1891 had required a guarantee with regard to the efficiency of the system, with deductions to be made for each percent of efficiency guaranteed but not achieved. The Westinghouse Company had guaranteed 88 percent. Comparison of meter readings at the two ends of the line showed a net transmission efficiency of 88.4 percent.

Future work was pretty much split between the two great manufacturers. The next seven 5000-horsepower generators (making ten in all in Power House No. 1) were by Westinghouse (see figures 10 and 11). In 1900, General Electric got the contract for the eleven 5500-horsepower generators in Power House No. 2. Beginning in 1905, the Niagara Falls Power Company also built and operated a station on the Canadian side. The first five generators, rated at 10,000-horsepower each, were by General Electric, and the next five, rated at 12,500-horsepower each, were by the Canadian Westinghouse Company, Ltd. All the power from the Canadian station was transmitted to Buffalo, on overhead lines on the Canadian side of the river from Niagara Falls to Fort Erie and continuing across the river from Fort Erie to Buffalo.

17. Forbes suggested using a generator of 20,000 V with a frequency of 16.67 Hz that would then run an induction motor. Why did Westinghouse reject this idea?

18. In 1895, the Niagara Falls power plant had three 5000 horsepower generators. Convert the total power output to Watts. (1 hp = 745 W)

19. If the total height of the falls is 54 meters, and the flow rate is $84{,}760\,\frac{\mathrm{ft}^3}{\mathrm{s}} = 2289\ \mathrm{m}^3/\mathrm{s}$, how much power *could* be generated? (Hint: How much potential energy is there in the falls?) (1 m^3 = 1000 kg)

20. In the end, there were ten 5000 hp generators, eleven 5500 hp generators, five, 10,000 hp generators, and five 12,500 hp generators. How does this total output compare the potential energy you calculated above?

The Birth of an Industry

The irony is, once the system was in operation, transmission to Buffalo, which in the planning stages was thought to be critical to the viability of the project, turned out to be much less of a factor. Primarily because the availability of abundant cheap power spawned an entirely new industry in Niagara Falls--the electrochemical industry--whose power requirements right from the start accounted for virtually the entire supply. The new industry spurred the production of greater quantities of electrical power, which in turn spurred further industrial development in Niagara Falls and all of Western New York.

…

In Buffalo the electrical power was used in grain handling and processing, iron foundries, machine shops. Through the first half of the twentieth century, the biggest industries in Buffalo were grain milling, iron and steel production, and all varieties of manufacturing from hairpins to airplanes.

Subsequent power production facilities would supplement and replace the original facilities. The present plant on the American side, operated by the New York State Power Authority, provides about 2.5 million kilowatts (which in the twentieth century came to replace the system of measurement in horsepower, 2.5 million kilowatts translating to 3.3 million horsepower). The plant on the Canadian side, operated by Ontario Hydropower, produces another 1.8 million kilowatts. The electricity is transmitted at 345,000 volts over a power grid that covers the nation and the world and is the ultimate legacy of the Niagara innovations.

21. Why did so many electrochemical companies relocate to Buffalo after the Niagara power plant was constructed?

Figure 2: Adam Beck Hydroelectric Power Station in Ontario, Canada (Image credit: [Adam Beck])

22. Today, the power output at Niagara Falls is 4.9×10^9 W (the article is out of date). How does this compare to the power generated by traditional coal or natural gas power plants?

23. At night, the power company uses the generator to pump water into a reservoir through a process called "hydro-pumping" (see the reservoir in Figure 2). This is effectively an energy storage system, like a battery. Why would the power company want to do this?

24. Compare the total power output to what you calculated in #19. What accounts for the difference?

25. Hydropower plants can by up to 95% efficient. How does this compare to a traditional power plant? What accounts for the difference?

26. The New York State Power Authority is generating 2.5 million kilowatts of electricity and transmitting it at a voltage of 345,000 V. What is the current in the transmission lines?

Project: Design a Tiny Generator

In the project for this unit, your group will be acting as an electrical engineering team responsible for designing a tiny generator. In short, a generator converts kinetic energy into electrical energy. In this unit, we have explored the following questions:

- How can we generate magnetic fields without a magnet?
- How can we generate current through a wire without using a battery?
- How does the electric grid in the US work to get the electricity to our homes?

Your task for this project is to build a simple generator, designed to particular specifications. You will be assigned a target voltage, and will need to use your newfound understanding of electromagnetism to determine the physical parameters of the generator.

Step 1: Mathematical Modeling

Earlier in this unit, we conducted several experiments on simple generators. We used the results of these experiments to develop an empirical model to describe the voltage generated. We also learned about Faraday's Law, and derived a theoretical model that can predict how much voltage is generated.

Empirical model: $V(t) = V_{max} \sin \omega t$ $\quad$ $V_{max} \propto N\omega B$

Faraday's Law: $V(t) = NBA\omega \sin \omega t$

1. What parameter(s) can we control and accurately measure when designing the tiny generator?

2. What parameter(s) cannot be easily measured? Explain.

Step 2: Measure Effective Magnetic Field

Physicists and engineers are often faced with the fact that they cannot accurate measure all of the parameters in their system. In these cases, they can develop what is called a **semi-empirical model**; that is, a model that is based partly on theory and partly on experimental data.

In the case of our generators, the magnetic field cannot be easily measured for a variety of reasons. (You should think about why for your report.) Before we can make an accurate prediction using Faraday's Law, we need to have a value to plug in for magnetic field. To do this, we will measure the **effective magnetic field** experimentally. This is a way of calibrating our model.

3. Pick any one of the generators you built previously. Use the Capstone software to measure the maximum voltage produced for a given rotational speed ω. Given this data, calculate the effective magnetic field strength of the magnets. (Hint: What other information will you need from your generator?)

4. How is the effective magnetic field different from the actual magnetic field?

Step 3: Design a Generator

5. Given the voltage output assigned to you, design a generator. How many turns will you need? What is the frequency of rotation? Use the experiments you did previously to help you choose reasonable values as a starting place.

6. Build a generator using the values you found above.
7. Test your generator and measure the maximum voltage produced.
8. How does the actual voltage generated compare to your prediction based on the mathematical model? What could account for any discrepancies?

9. What assumptions and/or approximations did you make that might need to be revisited?

10. What determines how much current is produced by the generator? How could you change the current in the system?

11. If your generator is not generating the amount of voltage you would like, make some changes to the system until it produces the desired voltage.

Written Report

Now that you have designed your tiny generator, your engineering team must document the design so that it can be manufactured for use in tiny houses. Your report must include the following components:

- Background Information
    - Why is it important to study generators and electromagnetism? What are the applications?
    - Explain how a generator is able to convert mechanical energy into electrical energy.
- Mathematical Model
    - Briefly explain the results of the Generator Lab and describe how you arrived at the empirical model.
    - Explain how you would, theoretically, be able to generate the voltage assigned to you using Faraday's Law of Induction. Show all calculations.
- Results of Calibration Test
    - What is the effective magnetic field of the system?
    - How can you use this information to design your generator?
- Specifications for the generator
    - Explain how you determined the specifications for your generator, including: Number of turns of wire, Strength of magnetic fields, and Rotation speed
- Conclusions
    - What assumptions and/or approximations that you made in the mathematical model held for the real generator?
    - What are the limits of the mathematical model? How did you address these in the design of your system?

Generator Project Rubric

Criteria	Points earned	Points possible
Introduction • Explain the applications of generators and why we would want to study them. • Explain the physics of how a generator works to convert mechanical energy into electrical energy.		10
Mathematical model • Explain how you would, theoretically, be able to generate the current and voltage assigned to you using Faraday's Law of Induction. Show all calculations. • If you based your prediction on empirical data, be sure to present all data and explain your reasoning. • Explain how and why the effective magnetic field was calculated.		10
Generator Testing • Specify the number of turns of wire, rotation speed, and strength of magnetic field. • Was the generator able to produce the required voltage? • How does this compare to your prediction? • What additional testing did you do? • What changes did you make?		10
Conclusions • Compare the results from your test to the predictions from your model. • What assumptions and/or approximations that you made in the mathematical model held for the real generator? • What are the limits of the mathematical model? How did you address these in the design of your system?		10
Writing Mechanics • Paper is proofread and free of grammatical and spelling errors. • All references are cited in the text and in a bibliography. • Equations are numbered and written on a line separate from the text.		5
Physics Content • Explanations and evidence demonstrate an understanding of the physics content. • All physics concepts and mathematical equations are correct. • All calculations are correct, and include appropriate units.		5
TOTAL		50